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FORM-CLASS TAPER CURVES AND VOLUME TABLES AND THEIR APPLICATION[1]

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I. GENERALIZED TAPER CURVES AND VOLUME TABLES

NEED FOR IMPROVEMENT OF TREE VOLUME TABLES

As an aid in determining the contents of stands of timber in timber cruising and growth studies, foresters are accustomed to use volume tables showing the average volume in a given unit of product of trees of different diameters and heights. These tables have generally been made up for each individual tree species, and for species occurring over a wide range of territory different sets of tables have often been constructed in different portions of their range. It is desirable to have tables which will be applicable over as wide a range as possible in order to eliminate the necessity of special studies for the construction of tables from local material for each individual forest or project. In the construction of tables for general application measurements have been made on large numbers of trees of a given species covering every possible condition in many localities and these have all been averaged together to give a single average volume for each diameter and height class. In spite of the enormous amount of work necessary for the construction of such tables, experience has shown that they are often not dependable for timber cruising nor for the more exacting demands of growth and yield studies. The work reported in this paper was commenced by the author at the School of Forestry, University of Idaho, in 1920, and the principal findings (*3*)[3] were reached at that institution before the author became connected with the United States Department of Agriculture, Forest Service, at the Northeastern Forest Experiment Station, in the fall of 1923. Under the auspices of the Forest Service the work was amplified and elaborated in preparation for its publication. The report is, therefore, a cooperative contribution from these two institutions.

The volume of a tree is dependent upon three factors—diameter, height, and form. Volume tables of the conventional type described above recognize only differences according to diameter and height and show volumes corresponding to the average form of the material

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[3] Reference is made by number (italic) to "Literature cited," p. 743.

from which they are constructed. Unless the form of the timber to be measured happens to average the same as the form of the trees from which the volume table was constructed it is evident that the table will not give a correct estimate of the volume of the timber. In attempting to include material from all possible conditions in the construction of volume tables the object is to get figures representing the true average form of the species in question, but in application, due to differences in density, age, site, or other factors, the timber over large areas may vary widely in form from the true average for the species and thus reduce the dependability of the tables for any particular stand. It has been recognized that differences in average form of the timber due to such local factors as those mentioned may cause more variation in volume for a given size class than a difference of a thousand miles in range.

In the construction of taper curves, which show the average diameter at any point on the stem for trees of different diameter and height classes, the same difficulty is encountered. Charts are constructed which attempt to show the average taper of the trees for any size class but no consideration is given to the fact that the actual tapers in a given stand may differ widely from this average.

In order to get dependable results in timber estimating, under all conditions, tables are needed which take into consideration the factor of form. In studying the growth of young timber it is found that volume tables made up from measurements of trees in virgin stands do not give reliable results. Tables differing from those for forest-grown material may also be necessary for timber grown in open old fields. In some species at least the growth of trees left after logging is known to be distributed over the stem in a manner differing from that in the original stands, and thus volume tables based on measurements of trees in the original stand may not be reliable in measuring the development of the residual crop. All these difficulties in the application of the conventional volume tables based on diameter and height alone are due primarily to differences in average form and may be more effectively met by a system which recognizes the factor of form in the basic tables.

The factor of form may be considered as the product of three components. In the first place, the shape of the main portion of the stem above the butt swell must be considered by itself. This is expressed by the type of form curve exhibited and is usually described under the categories of neiloidal, conical, or paraboloidal, although it will be shown that trees more generally approximate a hyperboloid in shape.

In the second place, trees vary in rate of taper independent of the type of form curve they exhibit. This is expressed by differences in form quotient or absolute form factors, and timber is usually described in respect to this component as full-boled or rapid in taper.

Finally, trees differ markedly in bark thickness and amount and upward extension of butt swell. This is important in timber estimating because the accepted standard for measuring standing timber is the diameter outside bark at breast height. For trees of the same shape and rate of taper in the main portion of the stem, the wood volume will vary for a given breast-high diameter, according to the thickness of bark and amount of butt swell at breast height.

DEVELOPMENT OF THE FORM-CLASS SYSTEM IN SWEDEN

In Sweden, form-class volume and taper tables have for a number of years been used as the basis for timber estimating. In these tables the trees are classified not only by diameter and height but also by form quotient, which is defined as the ratio between the diameter at one-half the height above breast height and the diameter at breast height. The form quotient, originally proposed by Schiffel (*25*) and later modified by Jonson (*13*) is the best expression of tree form yet developed.

Previous to Jonson's work, Schiffel (*25*) and Maass (*18*, *19*) had demonstrated that for trees of a given height and form class, taper was independent of diameter. In 1910 and 1911 Tor Jonson, working with Norway spruce (*Picea excelsa*) and Scotch pine (*Pinus sylvestris*), showed that taper for trees of a given form class was independent of height as well as diameter. His work also indicated that different species of conifers, and possibly of hardwoods as well, might be measured by the same set of generalized taper curves, since it is reasonable to assume that all species taper in accordance with the same fundamental growth principle and that many species might express this in similar manner.

The form-class taper and volume tables now in general use in Sweden were computed by Tor Jonson from a formula developed by A. G. Höjer in 1903 (*10*) from measurements of Norway spruce. This formula gives the diameter at any point on the stem in percentage of diameter at breast height. It has the form $\frac{d}{D} = C \log \frac{c+l}{c}$ in which D is the diameter at breast height, d is the diameter at distance l from the tip, and C and c are constants varying with the form quotient of the timber. As l is expressed in percentage of the total height above breast height, the equation is general and applicable to trees of any size class.

Tor Jonson (*13*, *14*) showed that Höjer's equation was in close conformity to the actual tapers of Norway spruce, but in comparing it with the actual tapers of Scotch pine he noted a considerable falling off in the tops, which he attempted to correct by the introduction of a so-called "biologic" constant in the original equation, making it read $\frac{d}{D} = C \log \frac{c+l-2.5}{c}$ for Scotch pine. In spite of this failure of Scotch pine to conform exactly to the original equation, Jonson decided that the deviation was not so serious but that the volume tables would be sufficiently accurate for this species as well as Norway spruce.

In using the form-class tables no attempt is made to estimate the form of each individual tree, but an estimate is made of the average form of the stand and the table for the corresponding form class is used for all the trees in the stand. The use of a single average form quotient for the entire stand is based on the proposition that most of the trees in the stand will have very nearly this average form and that the trees having a better form will be offset by those having a poor form. The validity of this proposition is a subject for investigation.

It is difficult to determine the average form quotient of the standing timber by actual measurement of the middle diameters of the

trees, but by the use of the form-point idea Jonson (*15*) developed an indirect method of estimating the average form of the timber. As a basis for the form-point method, Jonson assumed that the wood is laid on according to the stress to which the stem is subjected in supporting the crown, and that the wind, exerting its pressure against the crown, is the chief agent in setting up stresses in the stem. The distribution of the stresses in the different portions of the stem, which results in differences in form of the trees, therefore, depends upon the size, shape, and position of the crown. The form point is defined as that point in the crown where the wind pressure may be considered as concentrated, which usually will correspond to the center of gravity of the area presented to the prevailing wind. The relative height of the form point in the tree is thus an index of the length, size, and shape of the crown, and therefore an index of the form of the stem. The form-point height is expressed as percentage of the total height of the tree and is measured very rapidly with the use of a Christen hypsometer divided into 10 equal divisions. Jonson worked out a table showing the form quotient corresponding to different form-point heights for Norway spruce of different total height classes. He also found that density is a good indicator of the average form of the stand, and suggests that with a little practice the average form of a stand can be judged very well on the basis of its density.

Although Jonson's form-point idea is apparently giving satisfactory results in practical use in Sweden, the wind-pressure theory upon which it is based is by no means universally accepted. Hohenadl (*11*) presents convincing argument for the theory that the tree stem is built up in such a manner as to best support its own weight rather than to resist lateral bending from wind pressure. According to Hohenadl the diameter at any point on the stem is a function of the weight of the tree above that point and the mechanical strength of the section at the point in question. The form-point idea might be adapted to this conception just as well as to the wind-pressure theory. For trees of average development there would be little difference in the relative height of the form point according to the two theories. The fact that little difficulty has been experienced in the use of the form point in Sweden does not, therefore, either prove or disprove the correctness of Jonson's conception.

Another factor which enters into the application of these tables is the effect of the bark thickness and butt swell extending above breast height upon the form quotient of the timber. The formula used in computing the tapers applies for diameters inside bark and takes no account of any inflection of the stem curve toward the base of the tree which results whenever butt swell extends above breast height. The basis for application of volume tables is commonly the diameter outside bark at breast height so that different form quotients as well as different volumes will be indicated for the same d. b. h.[4] class when there are differences in thickness of bark or amount of root swell at the breast height point.

For the thin-barked Norway spruce, Jonson (*13*) found the form outside bark to be essentially the same as the form inside bark; but for Scotch pine (*14*) he found great differences in bark thickness

[4] D. b. h.=diameter at breast height.

in the timber from different portions of the country, which made the form outside bark poorer in varying degrees than the form inside bark. He also noted that when the root swell extended above breast height the indicated form quotient was lower than the actual form of the tree, but apparently this distortion was not general and Jonson suggests no satisfactory method of dealing with it.

Mattsson (*20*) has made a detailed study of the form and form variations of European and Siberian larch. He found their tapers almost identical and very similar to the form of Scotch pine as worked out by Jonson, although he noted a falling off in the upper sections of the tall trees in the larger form classes. The bark thickness was found to vary greatly in different stands but these differences could not be correlated with any fixed character of the stand but rather seemed to be due to heredity or a specialization of the species. Investigations in growing trees from seed from different countries substantiated this theory.

Mattsson found that the butt swell in larch reaches breast height in trees 10 meters tall and mounts steadily with increasing height. He reached no definite conclusions as to the factors affecting butt swell. He found the total range of form class within a stand to be 25 to 27 form class units, but the trees distributed themselves about the average in accordance with the normal probability curve. The variation of the form class within a stand could not be connected with any special character of the stand nor with the diameters of the trees. It was found that the variation of the form quotient in trees of the average diameter class was about as great as the total variation in form quotient in the entire stand.

Mattsson found a close correlation between average height of the stand and average form class, and in the few cases where estimates of the average form according to average height of the stand were not satisfactory the deviations seemed to be correlated with the density. He concluded that the average form class of the stand could be used with plenty of accuracy for practical work if it is judged on the basis of density and height or on height alone. Estimation of the average form class in this way gives more certain results than by the measurement of a number of sample trees unless the number of sample trees is more than forty or fifty.

From studies of form in fully stocked pine stands (*21*) Mattsson also concluded that a single average form class could be used for the entire stand. In this case he stated that the average form could be based on direct measurement of a number of sample trees representing a range of sizes in the stand. The variation interval of the form class within these stands was found to be 24 form-class units or about the same as for larch. Since in the fully stocked stands of pine the form class was found to increase with age, a table giving the average form class at different ages for stands of different densities is suggested for practical use in approximate estimation of the stand. From a study of the use of the form point for determining the average form class of the stand Mattsson found no difficulty in determining the form point in pine forests. He found that the difference in the estimate of form point by different persons was not more than 1 or 2 per cent. He concluded that with Jonson's form-point table the form class is generally estimated too low by about two form-class units. Petrini (*23*) found that Jonson's form-point function gives

an estimate of form class of Norway spruce which is too low by exactly the same amount. Both Mattsson and Petrini emphasized the fact that the form point gives the stand's average form class and not the form class of the individual stem.

Petrini (*24*) investigated the factors affecting the accuracy of form-class methods in Lapland pine. He found the bark thickness increasing directly with the diameter of the trees, the ratio between bark thickness and diameter breast high being 11.4 per cent. This relation appeared to be constant in all stands covered by the investigation. The root swell extended above 10 per cent of the trees' height in some instances and seldom averaged more than 3 per cent of the diameter breast high in any stand. It begins to affect the breast-height diameter in trees about 7 inches in diameter and increases rapidly with increasing diameter of the trees. Jonson's form-point function gave form-class values for pine running as much as 6.5 form-class units below the true form of the material.

In the fall of 1920 the writer began to investigate the applicability of the Swedish form-class system to American species. In the present paper it will be shown that many American commercial conifers are essentially alike in their taper in the main portion of the stem, and that the generalized equation of the stem curve can be expressed more exactly by a much simpler formula than that used in Sweden. From this it follows that if suitable methods of allowing for bark thickness and butt swell and of estimating the average form of timber in the field are developed, a single basic set of form-class taper curves and volume tables can be used for all species known to conform to the formula.

DEVELOPMENT OF NEW FORMULA FOR THE STEM CURVE FROM WESTERN YELLOW PINE MATERIAL

Measurements of the taper of 201 western yellow pines (*Pinus ponderosa*) in north Idaho were collected as a basis for testing the conformity of this species to Höjer's (*10*) equation, upon which the Swedish volume tables are based. This material included trees ranging in age from 30 years to overmaturity, and in size from 4 inches to 44 inches d. b. h. Trees were selected from stands of various types and on various sites. Very open grown as well as normal and overcrowded trees were included. In taking these measurements the breast-height point was first carefully located. Then the height from this point to the tip was measured with a tape, great care being taken to locate and measure the true tip whenever this had been broken off by the fall of the tree. The average diameter outside bark at the stump, breast height, and each tenth of the stem above breast height was then determined by measuring in two directions with a caliper. At each point of measurement the average bark thickness was measured by chopping into the tree on opposite sides of the stem.

The taper of each tree was then plotted, height above ground on diameter inside bark, in order to study the root swell and eliminate any effect it might have on the breast-height diameter. Wherever the reverse curve of the root swell extended above breast height the normal convex curve of the upper portion of the stem was continued downward to breast height, and from this was read what is called the "normal" diameter of the tree. The accuracy with which this

prolongation of the stem curve is accomplished is subject to a graphic check in the averages as described on page 736. This check can be applied to the first few stems plotted to serve as a guide for handling this part of the procedure.

Each of the upper diameters was then divided by this normal diameter with the slide rule, thus expressing the tapers of the tree in general terms of percentage of normal breast-height diameter for different percentages of the height above breast height. In this form the tapers of trees of all diameters and heights can be averaged together on the basis of form and the results compared to the values given by a generalized equation of stem form. The trees were grouped into form classes on the basis of their "normal" form quotient, as contrasted to the actual form quotient which would be obtained without correction for root swell. These average percentile tapers are presented in Table 1.

TABLE 1.—*Percentile tapers of western yellow pine*

Form class	Percentage of length from tip to breast height									Basis number of trees
	90	80	70	60	50	40	30	20	10	
	Percentage of "normal" d. i. b.[a] at breast height									
45	89.7	76.1	68.9	55.7	46.6	36.2	26.0	17.7	10.0	3
55	92.2	83.3	74.3	63.5	54.8	45.8	34.5	24.2	12.7	6
60	93.8	85.5	78.2	69.8	59.9	50.0	37.8	27.3	15.1	17
65	93.5	88.6	82.6	75.5	65.7	56.5	44.6	31.0	18.8	29
70	95.2	90.5	85.1	78.7	70.6	60.9	50.2	36.5	21.2	55
75	95.9	91.8	87.2	81.3	74.9	66.3	55.3	40.7	23.2	70
80	96.7	93.9	90.7	85.2	79.1	70.3	59.4	44.4	25.7	18
85	97.5	95.1	92.6	87.5	83.5	74.9	61.2	45.4	33.1	3

[a] D. i. b.=diameter inside bark.

The figures in Table 1 are the raw averages of the material, and it will be noted that direct comparison with Jonson's table of values from Höjer's equation for the different form classes is impossible because in no case does the raw average fall exactly on the class average to which it belongs. By harmonizing the raw averages through a system of plotting and replotting, values for the even form classes could be interpolated; but this is not necessary, as a more satisfactory comparison can be made from the raw averages by plotting them along with the values from Jonson's table as in Figure 1, with form classes as abscissae, upper diameters expressed as percentage of normal diameter breast high as ordinates, and a curve for each tenth of the stem above breast height. Values derived from a new equation, to be discussed presently, are also shown in Figure 1. It is evident from Figure 1 that in the lower form classes western yellow pine is quite consistent with Höjer's equation, but that in the larger form classes the diameters are much below the values from the equation in the upper portions of the stem and somewhat above them in the lower portions.

Jonson's figures for Scotch pine fell below Höjer's equation in the upper sections of the larger form classes in the same manner. His modification of the equation by the use of the "biologic" constant 2.5 gave him better results. This constant lowers the curve for the

section $x = 10$ in Figure 1 about three units in all form classes without materially changing its shape, and this effect so rapidly decreases in the sections below this that it is negligible in the lower half of the stem. In this way the values in the upper form classes are not reduced sufficiently and the values for the upper sections of the lower form classes (appearing in the lower left portion of the chart) are thrown too low. Increasing the biologic constant gives no better result.

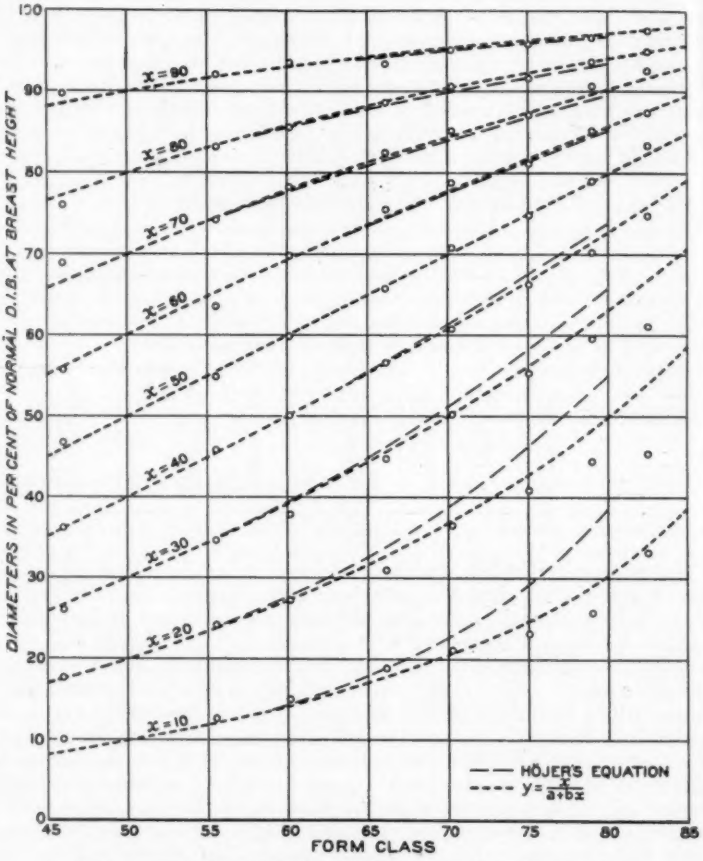

FIG. 1.—Percentile tapers of western yellow pine compared with formula values

When the biologic constant is 7.5 a good fit is obtained for form class 75 in all except the top section and form class 80 is somewhat improved, but the conformity of the lower form classes is entirely destroyed. The biologic constant makes a practically uniform reduction in the upper diameters of all form classes, whereas the variations noted in nature are confined to the higher form classes. Furthermore, increasing the value of the biologic constant exagger-

ates the inconsistency of the equation to a degree that can not be overlooked. This inconsistency lies in the fact that with the biologic constant the equation gives no diameter to that portion of the top of the tree represented by this constant. For small values like 2.5 per cent this is not serious, but the curve becomes entirely incompatible with nature if the value must be increased as high as 7.5 per cent.

Since the average values of the tapers of trees fall very nearly on a smooth curve, it seems probable that a relation does exist which may be represented mathematically by the equation of a curve, and if all trees lay on their growth in accordance with the same law such an equation ought to apply quite generally among different species. It seems evident that Höjer's equation fails to meet this requirement, as Norway spruce appears to be the only species approaching its value in the larger form classes. An attempt was therefore made to develop with the western yellow pine material an equation which would better express the normal taper of forest trees.

Graphic methods as presented by Lipka (*17*) were used in this analysis. The form of the curve of the experimental material gives some hint of the nature of the equation. It is desirable to choose an equation of the simplest possible form, the correctness of the form being tested by "rectifying" the curve, i. e., by rewriting the assumed equation in the form of a straight line in certain functions of the original variables. The data are then plotted with these functions of the original variables as coordinates; if the points of this plot appear to be on, or very near a straight line, then this line can be represented by the "rectified" equation, and hence the original curve by the assumed equation. When satisfied with the form of the equation chosen, it is necessary to determine values for the constants or coefficients appearing in the equation. The results should then be checked by new experimental material.

TABLE 2.—*Ratio of percentage of length from tip to percentage of normal d. i. b.[a] at breast height=x/y; western yellow pine*

Form class	Percentage of length from tip to breast height=x								
	90	80	70	60	50	40	30	20	10
	Values of x/y								
45	100.3	105.1	101.6	107.7	107.3	110.5	115.4	113.0	100.0
55	97.6	96.0	94.2	94.5	91.2	87.3	87.0	82.6	78.7
60	95.9	93.6	89.5	86.0	83.5	80.0	79.4	73.3	66.2
65	96.3	90.3	84.7	79.5	76.1	70.8	67.3	64.5	53.2
70	94.5	88.4	82.3	76.2	70.8	65.7	59.8	54.8	47.2
75	93.8	87.1	80.3	73.8	66.8	60.3	54.2	49.1	43.1
80	93.1	85.2	77.2	70.4	63.2	56.9	50.5	45.0	38.9
85	92.3	84.1	75.6	68.6	59.9	53.4	49.0	40.1	30.2

[a] D. i. b.=Diameter inside bark.

After a number of trials with equations of other types, the equation of the form $y=\frac{x}{a+bx}$, which represents an hyperbola, was found to correspond quite well with the western yellow pine material. This equation may be written $x/y=a+bx$, whereupon it becomes an equation of the first degree in x/y and x, and plotting with x/y and x as

coordinates should give a straight line. In this equation x corresponds to l of Höjer's equation and y to d/D, a and b being constants varying with the form class. The variable y may, therefore, be defined as the ratio of the diameter at distance x from the tip to the

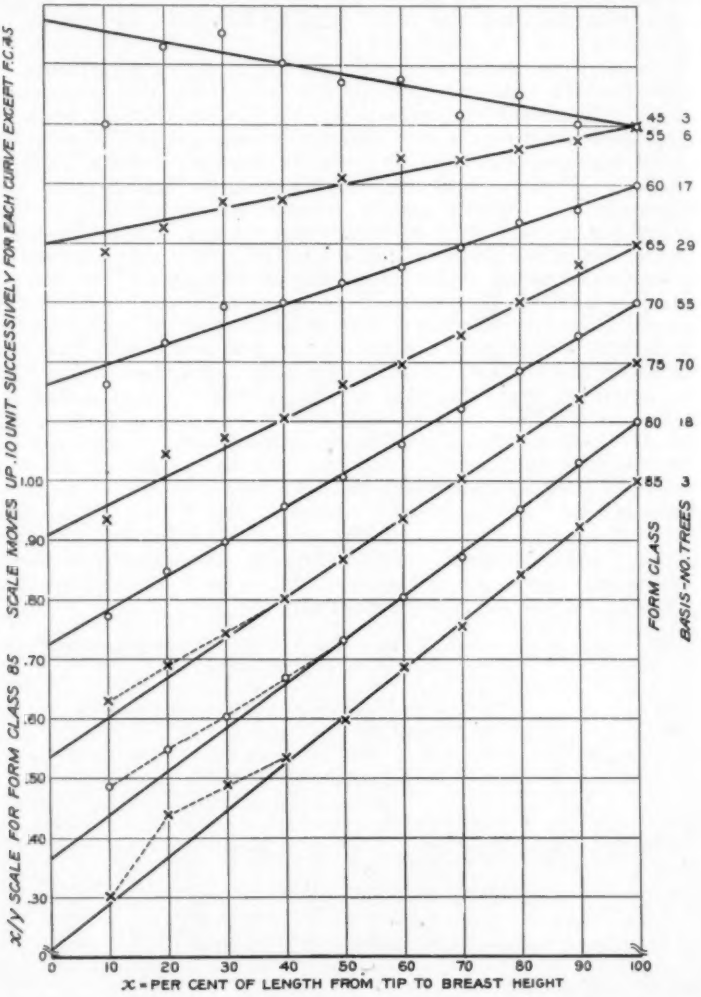

FIG. 2.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$, from data on western yellow pine, 201 trees, north Idaho

normal diameter at breast height, the distance x being expressed as percentage of the total height above breast height. Table 2 gives the corresponding values of x and x/y for the western yellow pine material as computed from the data in Table 1.

Figure 2 shows how nearly these values fall into straight lines when plotted. The scale of ordinates is moved downward 0.10 unit for each form class after 55 in order to keep the lines apart and present them more clearly. It will be noted that form classes 70 and 75, which have the best basis in number of trees, fit almost perfectly, and that the deviations in all the lower form classes are not consistent, but seem rather to indicate erratic averages, resulting from an insufficient number of trees. In form classes 80 and 85, however, there seems to be a definite tendency for the values of x/y for the upper sections to diverge upward from the straight line which fits the lower portion of the stem. This means that the actual diameters fall below the values given by the formula in these sections.

It should be emphasized that this equation, as well as that developed by Höjer, is entirely empirical, being an attempt to fit a curve to actual tapers as found in the field and is in no way dependent upon the fundamental theory of tree growth. No satisfactory formula has yet been proposed derived directly and deductively from the fundamental theories of tree growth.

TEST OF NEW EQUATION FOR OTHER SPECIES

With an equation satisfactorily representing the stem curve of western yellow pine, material for other species was examined to determine whether their taper could be measured by the same type equation. Table 3 summarizes data which were available for this test.

TABLE 3.—*Material used in comparative tests*

Species	Source and locality	Number of trees	Number of series
Norway spruce, *Picea excelsa* [a] [c]	Jonson, 1910, Sweden	47	3
Scotch pine, *Pinus sylvestris* [a]	Jonson, 1911, Sweden	94	4
European larch, *Larix europea* [b]	Mattsson, 1917, Sweden	503	12
Siberian larch, *Larix sibirica* [b]	....do	229	3
Western white pine, *Pinus monticola*, [b] [c]	Behre, 1923; U. S. F. S., 1910–1913, north Idaho	139	6
Do.[b]	Claughton-Wallin, and McVicker, 1920, British Columbia. (7)	16	1
Douglas fir, *Pseudotsuga taxifolia* [b]	U. S. F. S., 1910–1913, North Idaho	72	5
Do.[b]	Claughton-Wallin, and McVicker, 1920, British Columbia.	11	1
Norway pine, *Pinus resinosa* [c]	Claughton-Wallin, and McVicker, 1920, Ontario	81	3
Do.[b]	Reed, 1924, Massachusetts and New Hampshire	36	2
Northern white pine, *Pinus strobus*. [c]	Claughton-Wallin, and McVicker, 1920, Ontario (young stands).	29	3
Do.[a] [c] [d]	Wright, 1923, Ontario (old stands)	342	9
White, red, and black spruce [a] [c] [d]	Wright, 1923, Canada	865	8
White spruce, *Picea glauca* [c]	Claughton-Wallin, and McVicker, 1920, Ontario	6	1
Red spruce, *Picea rubra* [b]	U. S. F. S., 1902, Piscataquis Co., Me. (virgin stands).	132	4
Do.[b]	U. S. F. S., 1903, New Hampshire (second growth)	20	1
Do.[b]	U. S. F. S., 1903, New Hampshire (culled forest)	44	3
Do.[b]	U. S. F. S., 1903; Meyer, 1924, New Hampshire and Maine (old field).	92	6
Balsam fir, *Abies balsamea* [a] [c] [d]	Wright, 1923, Canada	362	4
Do.[b]	Meyer, 1923, Maine (second growth)	18	1
Do.[c]	Behre, 1924, Franklin Co., Me. (culled forest)	20	1
Do.[c]	Behre, 1924, Franklin Co., Me. (old growth)	73	5
Longleaf pine, *Pinus palustris* [a] [b]	Brown, 1924, southern United States	663	3
Shortleaf pine *Pinus echinata* [a] [b]	....do	416	3
Loblolly pine, *Pinus taeda* [a] [b]	....do	338	3
Gray birch, *Betula populifolia* [b]	Hawley, 1923, Pennsylvania	177	6
Eastern hemlock *Tsuga canadensis* [c]	Merrill and Hawley, 1923, Connecticut	55	3

[a] No attempt to eliminate root swell in the averages.
[b] Trees measured at arbitrary intervals in the field, diameters at one-tenth intervals interpolated graphically.
[c] Trees measured at one-tenth intervals in the field.
[d] Harmonized and interpolated to even form classes.

Figures 3 to 15 show the plotting of x/y on x for these data either taken directly from the various authors or interpolated from their figures.

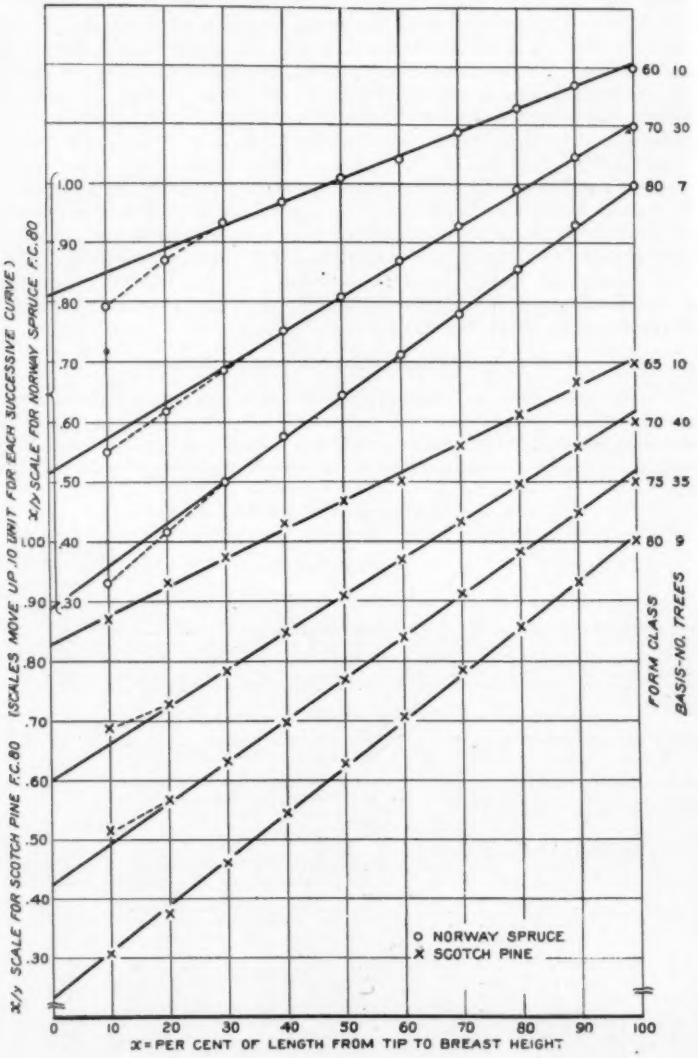
60 10
70 30
80 7
65 10
70 40
75 35
80 9
FORM CLASS
BASIS-NO. TREES
O NORWAY SPRUCE
X SCOTCH PINE
X/y SCALE FOR SCOTCH PINE F.C.80 (SCALES MOVE UP .10 UNIT FOR EACH SUCCESSIVE CURVE)
X/y SCALE FOR NORWAY SPRUCE F.C.80
X = PER CENT OF LENGTH FROM TIP TO BREAST HEIGHT
0 10 20 30 40 50 60 70 80 90 100
1.00 .90 .80 .70 .60 .50 .40 .30

FIG. 3.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$, from data on Norway spruce and Scotch pine as given by Tor Jonson

These charts reveal a striking similarity in the form of the various species with the exception of hemlock, and indicate, with this excep-

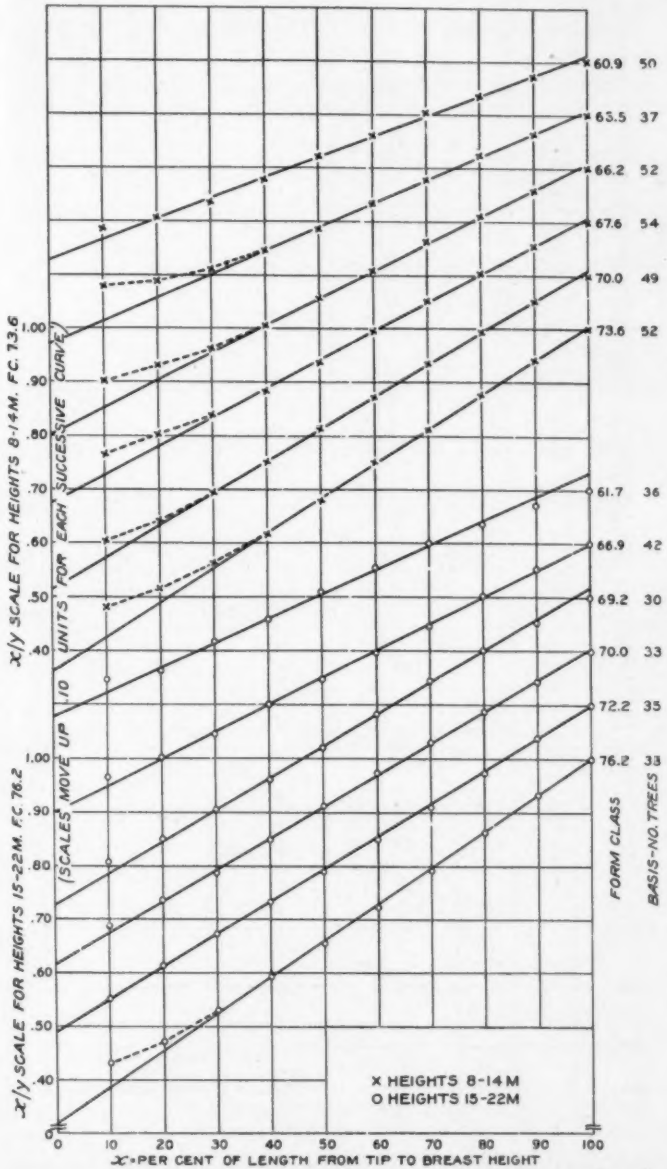

FIG. 4.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$, from data on European larch as given by L. Mattsson

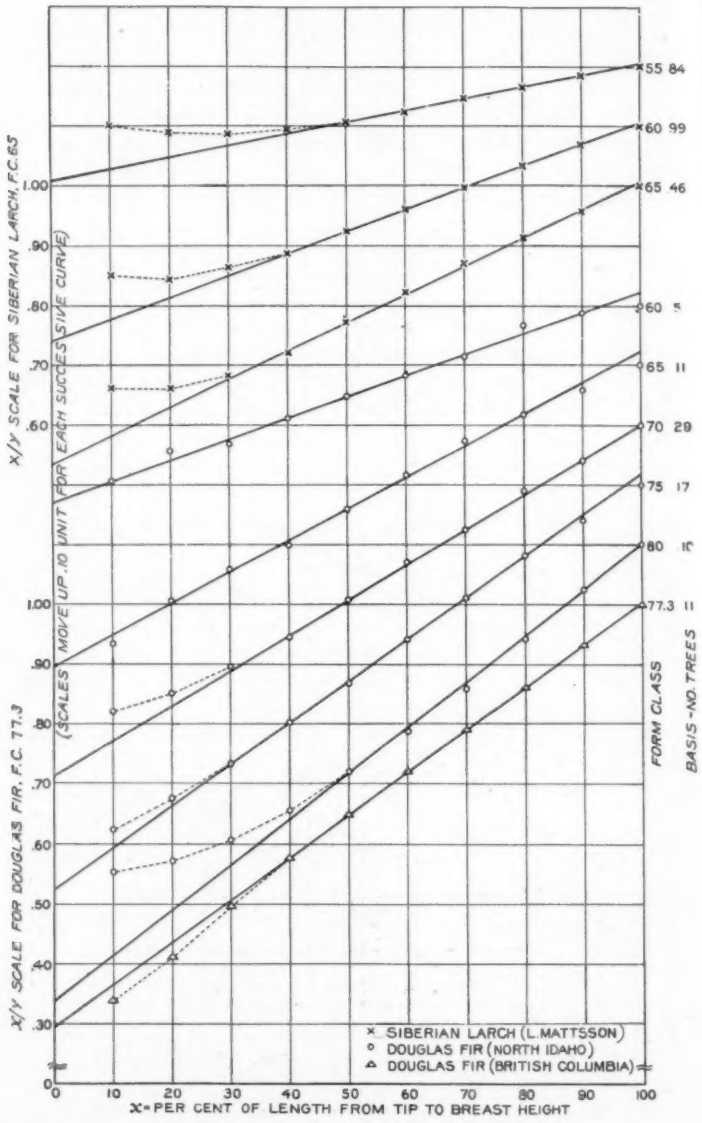
X/Y SCALE FOR SIBERIAN LARCH, F.C.65
(SCALES MOVE UP .10 UNIT FOR EACH SUCCESSIVE CURVE)
X/Y SCALE FOR DOUGLAS FIR, F.C. 77.3
FORM CLASS
BASIS - NO. TREES
55 84
60 99
65 46
60 5
65 11
70 29
75 17
80 10
77.3 11
× SIBERIAN LARCH (L. MATTSSON)
○ DOUGLAS FIR (NORTH IDAHO)
△ DOUGLAS FIR (BRITISH COLUMBIA)
X = PER CENT OF LENGTH FROM TIP TO BREAST HEIGHT

FIG. 5.—Plotting of x/y on x showing conformity to $y = \frac{x}{a+bx}$, from data on Siberian larch as given by L. Mattsson; Douglas fir, north Idaho; and Douglas fir, British Columbia

tion, a satisfactory conformity to the type equation $y=\frac{x}{a+bx}$. It will be noted, however, that in many of the species there is still a definite and constant tendency in some classes for the tops to fall below the values given by the formula, as indicated by the divergence

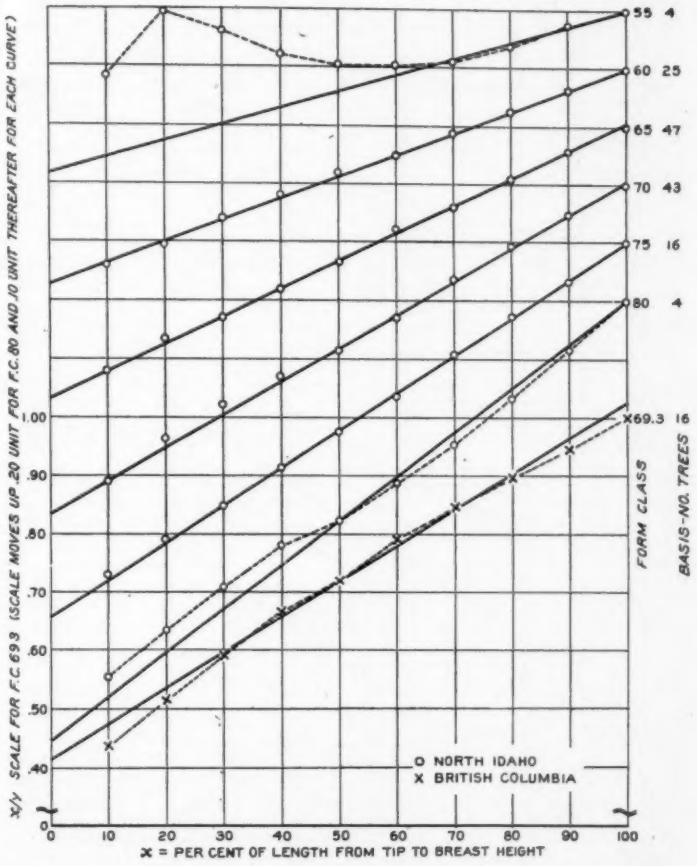

FIG. 6.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$, from data on western white pine in north Idaho and British Columbia

upward from the straight lines of the points for the upper sections. This divergence in the tops varies with the different species, but seems to be fairly constant within the species. It is practically negligible in all the species studied except hemlock and balsam fir up to form class 65 but increases rather rapidly above form class 70. The nature of this variation will be discussed later.

DIRECT COMPARISON OF SPECIES TAPERS

In order to bring all the material together for direct comparison the different series can be plotted on one chart by the method of

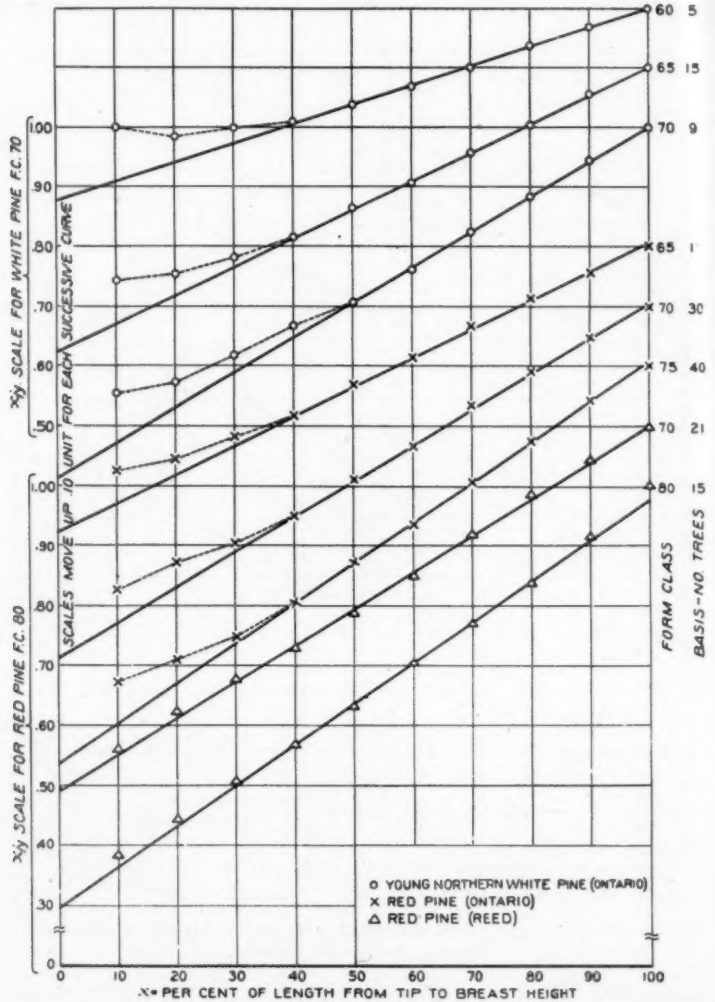

FIG. 7.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$, from data on young northern white pine in Ontario, red pine in Ontario, and red pine as given by Reed

Figure 1. In this chart form classes are abscissae, values of y are ordinates and a separate curve is drawn for each value of x. The values for any series, therefore, appear as a set of points on the

vertical line corresponding to its true form class. In order to determine the proper form-class line on which to plot a series it is necessary first to be sure that root swell is correctly eliminated from the raw

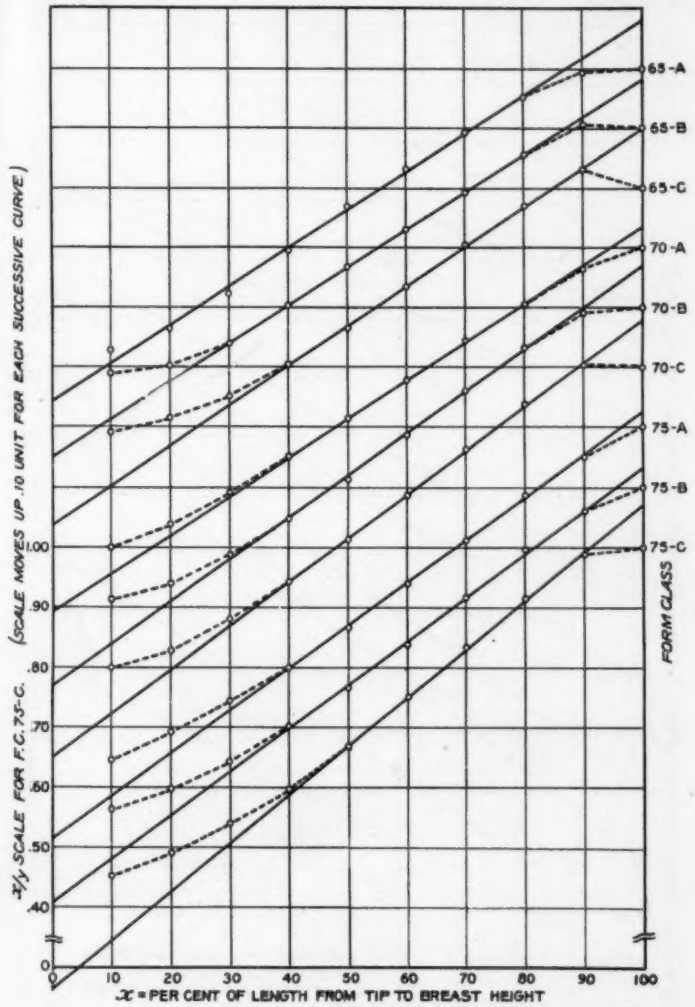

FIG. 8.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$, from data on northern white pine in Canada, as given by W. G. Wright

averages, and, second, to examine the diameter at half height to find out whether it is erratic as compared to the measurements of adjacent sections and if so to bring it into line with them. These corrections are necessary because the form quotient derived from the

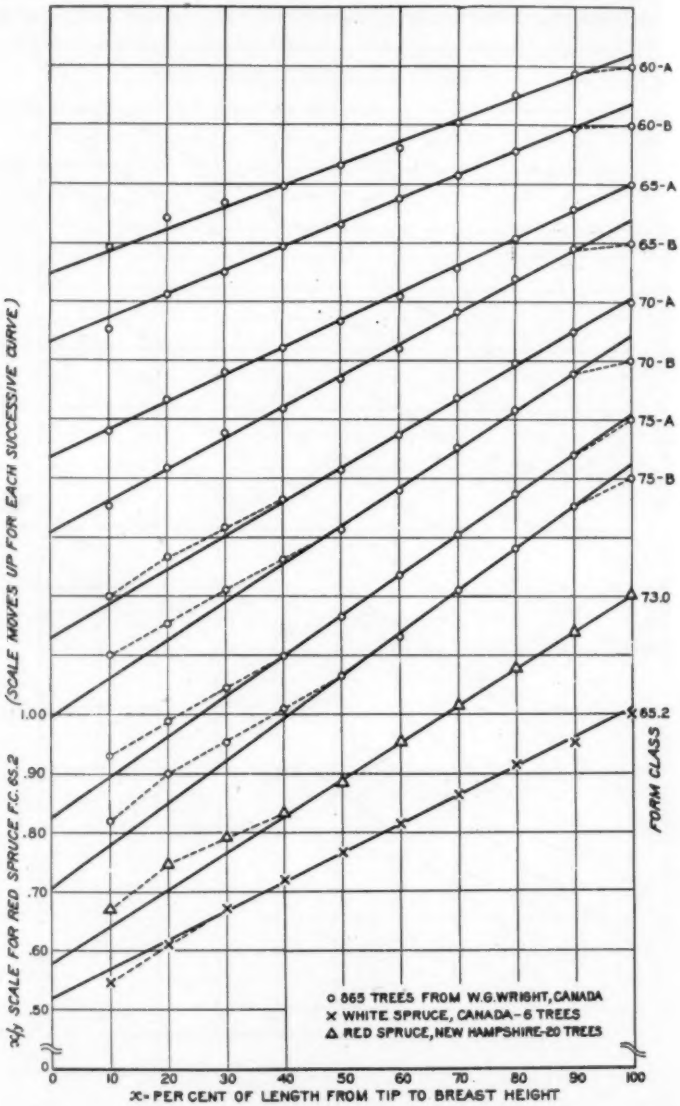

FIG. 9.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$, from data on red, white, and black spruce

raw averages may not indicate the form curve corresponding most closely to the series in question if the diameter at breast height is distorted by root swell, or if through the chance errors of sampling

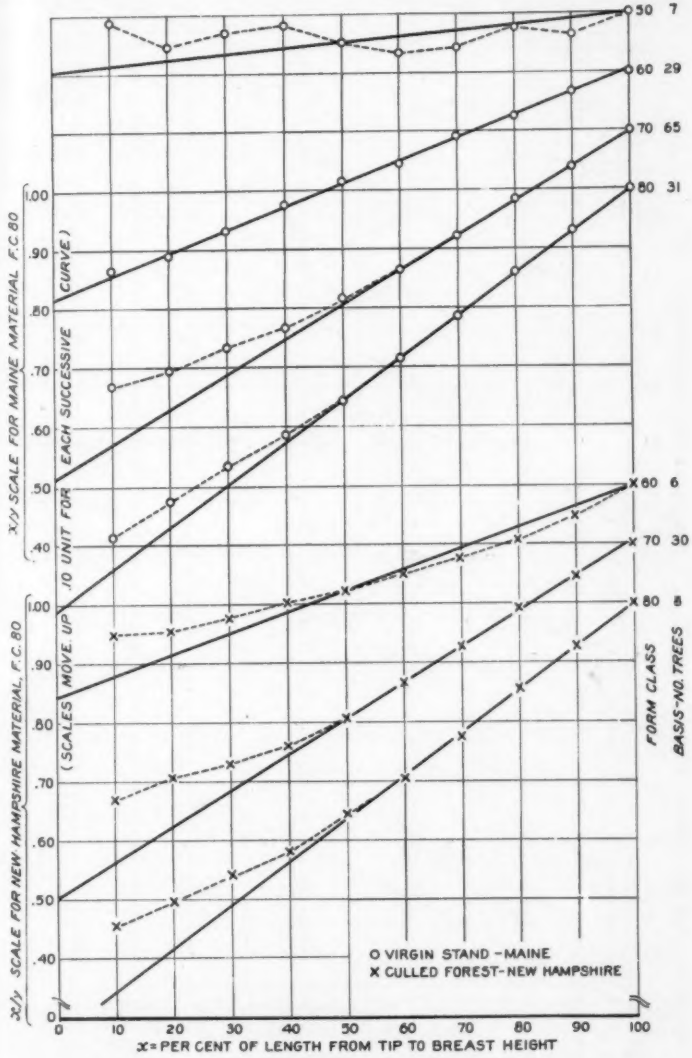
50 7
60 29
70 65
80 31
60 6
70 30
80 8
FORM CLASS
BASIS-NO. TREES
x/y SCALE FOR MAINE MATERIAL F.C. 80
(SCALES MOVE UP .10 UNIT FOR EACH SUCCESSIVE CURVE)
x/y SCALE FOR NEW HAMPSHIRE MATERIAL, F.C. 80
O VIRGIN STAND - MAINE
X CULLED FOREST - NEW HAMPSHIRE
x = PER CENT OF LENGTH FROM TIP TO BREAST HEIGHT

FIG. 10.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$, from data on red spruce

either the diameter at breast height or the diameter at the mid-point is out of line with adjacent sections. Both of these corrections can be made easily and satisfactorily by the use of the straight line test charts of x/y on x.

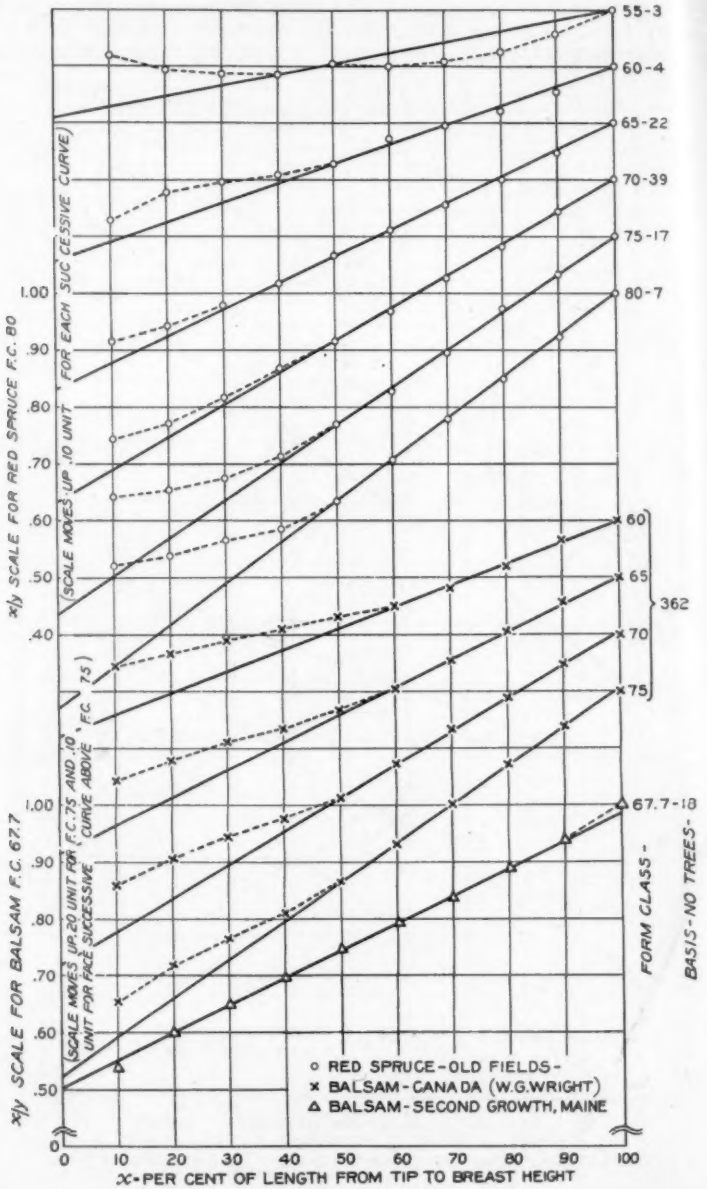

FIG. 11.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$, from data on old fields of red spruce; balsam fir in Canada, as given by W. G. Wright; and second-growth balsam fir in Maine

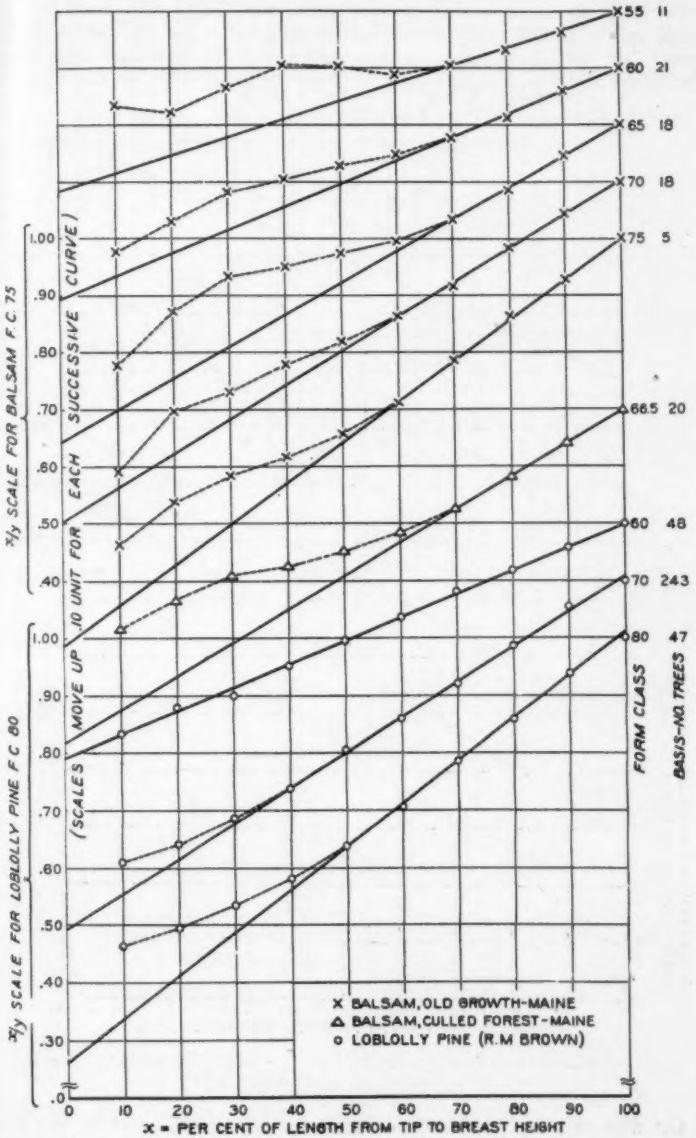

FIG. 12.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$; from data on old-growth and culled forest balsam fir in Maine, and loblolly pine as given by R. M. Brown

In Figures 2 to 14 it will be noted that in many of the series, especially those in which no attempt was made to eliminate root swell in the preliminary compilation, the value for the breast-height point ($x = 100$) falls considerably below the straight line best fitting the intermediate sections of the tree. This deviation is the result

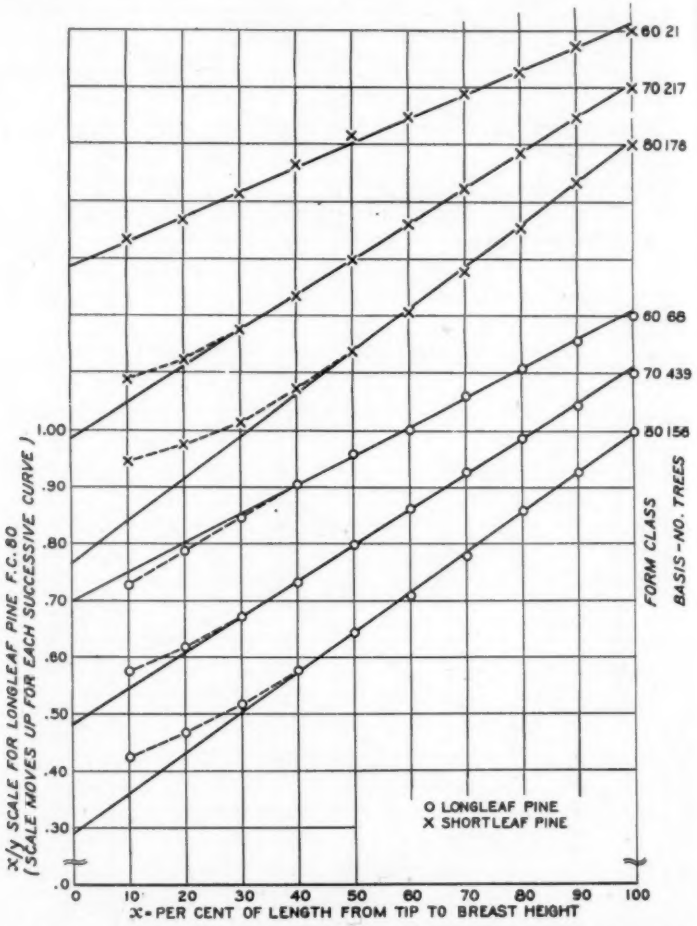

FIG. 13.—Plotting of x/y on x showing conformity to $y = \frac{x}{a+bx}$, from data on long-leaf and short-leaf pine as given by R. M. Brown

of the distortion of the normal curve by root swell, and so the true normal diameter at breast height can be calculated from the value of x/y read from the intersection of the best fitting straight line and its breast height abscissa. Thus, in Figure 3, Scotch pine F. C. 80, the straight line best fitting the plotted points in the intermediate

sections crosses the breast height abscissa at 1.012. The diameter to be used as the base for the true normal form quotient is, therefore, $\frac{100}{1.012}=98.8$. In a similar manner an examination of the figure shows that the value at half height ($x=50$) falls slightly above the line giving best fit to the series as a whole, indicating that this average may be considered somewhat erratic. The middle diameter

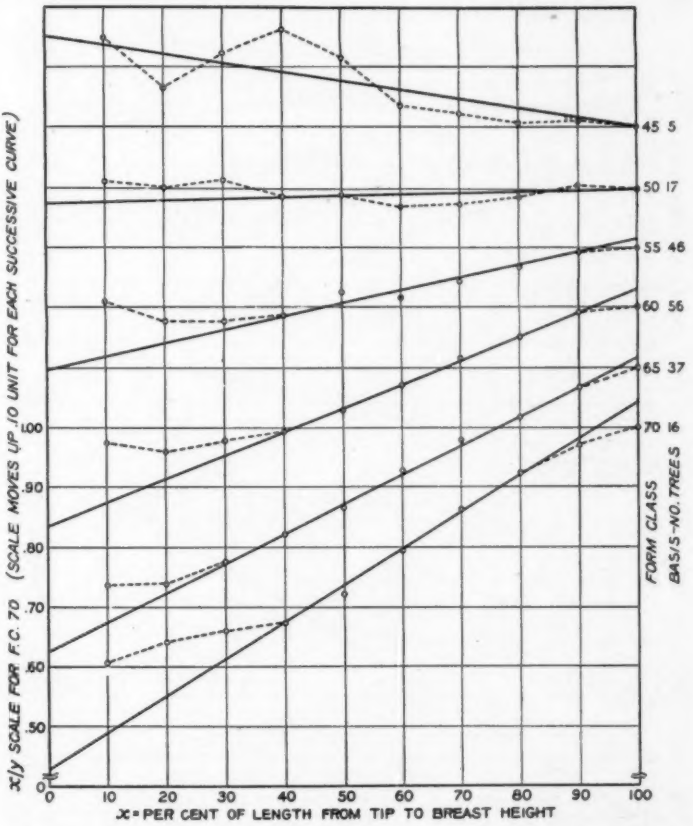

FIG. 14.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$, from data on gray birch as given by R. C. Hawley

to be used in calculating the true normal form quotient is determined from the intersection of the best fitting straight line with the abscissa for the midpoint. Thus, $x/y=.624$ and $\frac{50}{.624}=80.1$, the "normal" diameter at half height. The form class line upon which this series should be plotted in our comparative chart is, therefore, $\frac{80.1}{98.8}\times 100=81.1$ instead of 79.4 as indicated by the raw material.

This new value of the form quotient is more reliable than that obtained from the raw material because it is based on the fit of the entire stem rather than upon the arbitrary field measurement of the two points in question. This principle can be applied to those series in which root swell was eliminated graphically before averaging as well as to those in which the averaging was done directly from the

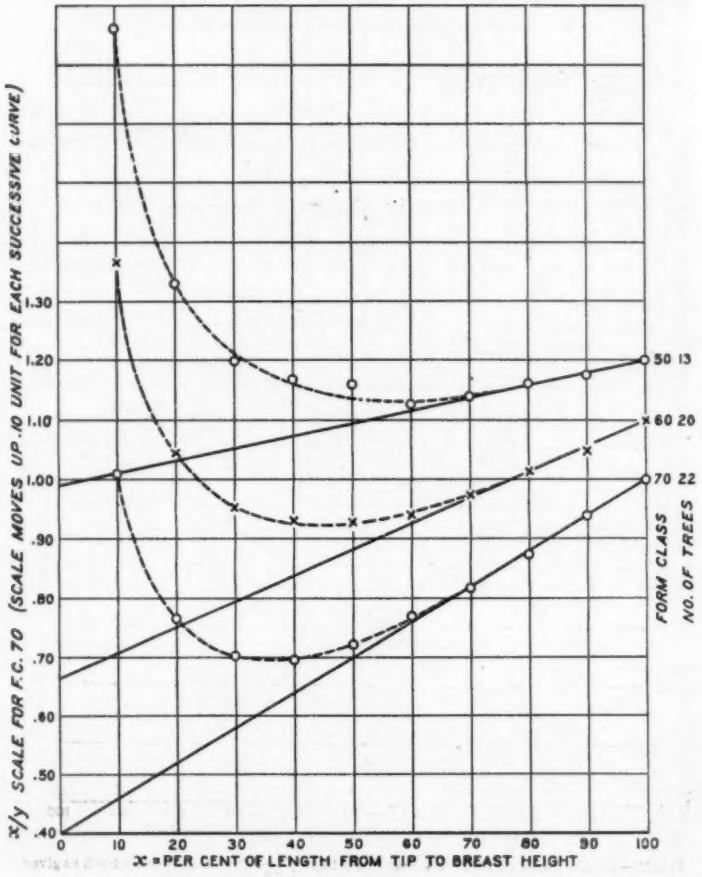

FIG. 15.—Plotting of x/y on x showing conformity to $y=\frac{x}{a+bx}$, from data on eastern hemlock

tree measurements, and serves as a check upon the graphic method. The straight-line test will reveal in the average any consistent tendency to exaggerate, or failure to make sufficient allowance for root swell in the plotting of the individual trees. In applying this principle to series where the divergence of the material from a straight line is marked, as with balsam, the straight line used to determine the normal form class is that best fitting the lower portions

of the stem, because only in this way can a direct comparison of the amount of divergence of the tops from the formula be obtained for the different species.

After determining the true normal form quotient for each series by the method just described, all the material with the exception of hemlock, which requires special treatment, has been brought together for direct comparison on a single chart. Whenever the diameter at breast height had been modified to eliminate root swell the remain-

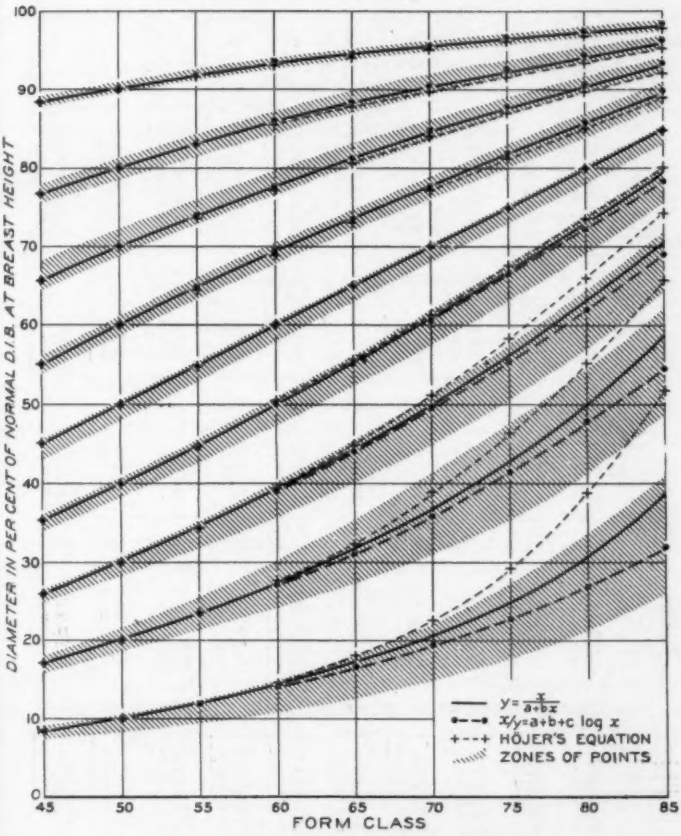

FIG. 16.—Range of percentile tapers compared with formulae

ing percentile tapers were recomputed with the true "normal" diameter as a base in order to compare them properly with other series. The data for this comparison are brought together in Table 4. Because of the large number of points grouped closely together it is difficult to identify the individual series in the plotting; but the points for the different sections fall into well-defined bands. Figure 16 shows the approximate upper and lower limits of the band of

points at each section together with the curves given by Höjer's equation, by the simple form $y=\frac{x}{a+bx}$, and by a modification of the latter to be discussed presently.

TABLE 4.—*Normal form quotients and adjusted percentile tapers*

Form class	Normal form quotient	Percentage of length from tip to breast height=x									Basis number of trees
		90	80	70	60	50	40	30	20	10	
		Adjusted percentile tapers									
WESTERN YELLOW PINE											
45	46.0	89.7	78.1	68.9	55.7	46.6	36.2	26.0	17.7	10.0	3
55	55.6	92.2	83.3	74.3	63.5	54.8	45.8	34.5	24.2	12.7	6
60	60.2	93.8	85.5	78.2	69.8	59.9	50.0	37.8	27.3	15.1	17
65	66.1	93.5	88.6	82.6	75.5	65.7	56.5	44.6	31.0	18.8	29
70	70.2	95.2	90.5	85.1	78.7	70.6	60.9	50.2	36.5	21.2	55
75	75.0	96.5	92.4	87.7	81.8	75.4	66.7	55.6	40.9	23.3	70
80	79.1	96.7	93.9	90.7	85.2	79.1	70.3	59.4	44.4	25.7	18
85	82.6	97.5	95.1	92.6	87.5	83.5	74.9	61.2	45.4	33.1	3
NORWAY SPRUCE (JONSON-1910)											
60	62.2	93.6	86.7	79.3	71.5	62.1	52.3	41.2	30.0	17.0	10
70	70.7	95.7	90.5	84.9	78.4	70.7	61.7	51.3	38.8	22.4	30
80	77.6	96.6	93.3	89.4	83.9	77.3	69.6	60.0	48.1	30.3	7
SCOTCH PINE (JONSON-1911)											
65	65.6	93.9	88.5	82.2	75.7	65.7	55.4	44.9	32.0	17.7	10
70	71.9	95.8	90.9	85.7	79.4	71.9	62.9	52.4	38.6	20.8	40
75	75.9	96.7	92.4	87.8	82.6	76.0	68.1	57.3	43.5	24.5	35
80	81.1	97.8	94.3	90.3	85.6	80.4	74.1	65.7	54.0	33.0	9
EUROPEAN LARCH—HEIGHTS 8 TO 14 METERS (MATTSSON-1917)											
60.9	61.8	94.0	86.6	78.8	70.8	61.9	52.3	41.5	28.8	14.8	50
63.5	63.9	94.0	87.4	80.5	72.7	64.0	54.0	42.4	29.2	14.8	37
66.2	66.8	94.9	88.9	82.2	75.1	66.9	57.4	45.9	32.0	16.8	52
67.6	67.9	95.0	89.2	83.0	76.1	68.2	58.8	47.2	33.4	17.8	54
70.0	70.8	95.8	90.6	84.9	78.5	70.9	62.0	51.1	37.2	20.1	49
73.6	73.4	96.2	91.5	86.6	80.2	74.0	65.1	53.7	38.7	20.8	52
EUROPEAN LARCH—HEIGHTS 15 TO 22 METERS (MATTSSON-1917)											
61.7	64.1	95.8	88.1	80.4	72.5	63.8	54.4	43.2	31.1	15.9	36
66.9	66.7	94.4	88.5	82.4	75.3	66.9	57.1	46.2	33.2	17.6	42
69.2	70.5	96.2	90.6	84.4	78.1	70.6	61.6	50.6	36.9	20.1	30
70.0	70.5	95.7	90.5	84.7	77.9	70.5	61.8	51.4	37.4	20.5	33
72.2	71.9	95.8	91.5	86.3	79.9	72.2	63.2	52.3	38.8	22.1	35
76.2	75.6	96.5	92.7	88.3	82.9	76.2	67.6	56.6	42.4	23.1	33
SIBERIAN LARCH (MATTSSON-1917)											
55	55.4	91.9	83.1	74.4	65.2	55.4	44.9	34.0	22.6	11.2	84
60	61.1	93.4	86.3	78.5	70.2	61.1	51.1	39.5	27.0	13.4	99
65	65.4	94.7	88.2	81.2	73.5	65.4	56.1	44.3	30.5	15.2	46

TABLE 4.—*Normal form quotients and adjusted percentile tapers*—Continued

WESTERN WHITE PINE (BEHRE-1923 AND U. S. F. S.-1910-1913)

Form class	Normal form quotient	Percentage of length from tip to breast height = x								Basis number of trees
		90	80	70	60	50	40	30	20	10	
		Adjusted percentile tapers									
55	58.2	92.4	85.2	76.8	66.4	55.2	43.3	31.1	20.1	11.3	4
60	61.4	93.4	86.2	78.8	70.5	60.8	51.1	40.4	28.7	15.1	25
65	65.7	94.3	88.4	81.4	73.4	65.4	55.5	44.7	31.4	17.3	47
70	69.7	95.3	89.9	84.0	78.0	70.0	59.9	48.3	35.6	20.4	43
75	73.7	96.2	91.5	86.4	81.2	74.0	65.3	54.5	40.7	23.2	16
80	80.2	98.0	95.8	92.8	87.3	80.2	68.8	58.7	46.1	28.1	4

WESTERN WHITE PINE (CLAUGHTON-WALLIN AND McVICKER-1920)

Average	71.2	97.6	91.7	84.9	77.8	71.2	61.8	52.2	39.8	23.3	16

DOUGLAS FIR (U. S. F. S.-1910-1913)

60	60.4	93.2	84.6	78.4	69.7	60.3	50.5	40.0	27.0	14.5	5
65	67.3	96.0	89.0	82.0	75.0	67.3	58.5	46.7	33.9	19.1	11
70	70.5	95.6	89.8	84.7	77.9	70.5	61.8	50.2	36.2	19.2	29
75	75.9	97.8	92.6	87.8	82.4	76.2	67.6	57.4	43.0	24.1	17
80	80.6	97.5	95.0	92.1	87.3	80.6	72.0	59.2	42.2	22.0	10

DOUGLAS FIR (CLAUGHTON-WALLIN AND McVICKER-1920)

Average	77.4	96.8	93.0	88.6	83.6	77.4	69.5	60.6	48.7	29.6	11

NORWAY PINE (CLAUGHTON-WALLIN AND McVICKER—1920)

65	65.8	94.7	88.2	81.3	74.3	65.5	56.1	44.2	31.2	16.1	11
70	70.8	95.5	90.4	84.7	78.6	70.7	61.8	49.9	35.2	19.1	30
75	75.0	96.3	92.2	87.3	82.1	75.0	66.5	55.1	39.4	21.4	40

NORWAY PINE (REED—1924)

70	71.8	95.4	90.5	85.6	80.0	72.6	63.6	51.8	38.2	21.7	21
80	77.0	96.3	93.4	88.7	83.3	77.2	68.9	58.1	44.6	25.2	15

YOUNG NORTHERN WHITE PINE (CLAUGHTON-WALLIN AND McVICKER—1920)

60	59.7	93.0	85.5	77.8	69.0	59.7	49.4	37.6	25.5	12.5	5
65	65.7	94.2	88.4	81.7	74.3	65.5	55.9	44.0	30.5	15.5	15
70	70.8	95.2	90.6	84.9	78.6	70.8	60.0	48.5	34.9	18.0	9

OLD NORTHERN WHITE PINE (WRIGHT—1923)

65-A	71.0	97.9	90.7	84.7	78.1	70.3	62.2	52.2	38.3	20.4	342
65-B	70.6	96.8	90.6	84.8	78.2	70.5	61.8	50.8	36.0	18.4	
65-C	71.5	96.0	90.7	85.1	79.1	71.6	62.3	50.7	35.8	18.6	
70-A	72.5	98.5	91.1	85.7	79.7	72.5	63.5	52.4	38.3	20.7	
70-B	74.2	97.1	91.8	87.0	81.6	74.9	66.1	54.6	39.6	20.8	
70-C	75.4	96.8	92.2	87.6	82.2	75.7	67.0	55.7	40.9	21.6	
75-A	76.4	97.0	92.5	88.2	83.2	76.9	68.1	56.4	41.5	22.9	
75-B	76.8	96.6	92.2	88.3	83.8	77.5	68.5	56.8	41.4	22.2	
75-C	80.2	97.6	93.8	89.9	85.8	80.5	71.9	59.2	43.6	23.6	

TABLE 4.—*Normal form quotients and adjusted percentile tapers*—Continued

WHITE, RED, AND BLACK SPRUCE (WRIGHT—1923)

Form class	Normal form quotient	Percentage of length from tip to breast height=x									Basis number of trees
		90	80	70	60	50	40	30	20	10	
		Adjusted percentile tapers									
60-A	61.1	93.1	85.9	79.0	71.2	61.3	51.1	39.9	27.6	14.7	865
60-B	62.1	93.7	86.7	79.1	71.0	62.2	52.1	41.3	29.0	15.8	
65-A	65.4	94.6	88.7	82.2	74.5	65.5	55.7	44.4	31.8	17.3	
65-B	67.1	94.6	88.7	82.5	76.1	67.6	58.1	46.7	33.7	18.7	
70-A	70.1	95.4	90.0	84.4	77.9	70.5	60.6	48.8	35.4	20.1	
70-B	72.3	95.6	90.7	85.4	79.9	72.8	d2.6	50.9	37.4	20.8	
75-A	75.4	96.4	92.2	87.7	82.3	75.8	67.2	55.4	41.2	23.4	
75-B	76.8	96.8	92.9	88.5	83.8	76.8	67.1	55.3	41.0	24.0	

WHITE SPRUCE (CLAUGHTON-WALLIN AND McVICKER—1920)

Average	65.9	95.3	88.4	81.5	74.1	65.9	56.1	45.2	33.1	18.4	6

VIRGIN RED SPRUCE (U. S. F. S.—1902)

50	52.4	93.2	81.7	74.1	64.1	52.4	40.6	30.9	21.1	10.1	7
60	62.0	93.6	86.8	79.0	71.2	61.7	51.7	41.1	29.1	15.1	29
70	70.9	95.8	90.4	84.8	78.2	69.8	59.9	47.2	33.6	17.5	65
80	77.8	96.6	93.0	89.0	84.0	77.6	68.1	56.0	42.0	24.2	31

RED SPRUCE-CULLLED FOREST (U. S. F. S.—1903)

60	60.8	94.8	88.2	79.8	70.6	60.8	49.9	38.6	26.5	13.3	6
70	71.4	96.0	90.6	85.2	79.0	71.2	60.8	47.9	33.2	17.7	30
80	78.5	96.8	93.4	90.1	85.0	77.3	68.6	55.1	40.0	21.9	8

SECOND-GROWTH RED SPRUCE (U. S. F. S.—1903)

Average	72.6	95.8	91.2	86.0	79.8	73.0	63.1	50.7	36.7	21.3	20

OLD-FIELD RED SPRUCE (U. S. F. S.—1903 AND MEYER—1924)

55	55.3	94.2	86.5	76.9	66.7	55.3	45.3	33.9	22.5	10.9	3
60	60.3	94.3	86.8	78.1	68.8	60.3	49.5	37.7	25.8	13.7	4
65	65.2	95.1	88.8	81.8	74.0	65.2	55.9	44.2	31.2	16.2	22
70	69.9	95.3	90.7	84.8	78.2	69.9	59.9	48.5	35.0	18.4	39
75	74.9	96.4	91.7	87.9	82.4	74.9	65.2	52.3	36.0	18.5	17
80	78.9	97.5	94.1	90.1	84.9	78.9	68.5	53.2	37.2	19.2	7

BALSAM FIR (WRIGHT—1923)

60	61.6	93.3	86.9	79.4	70.5	60.0	49.3	38.0	26.1	13.4	362
65	65.9	94.0	88.2	82.0	74.6	65.0	54.4	42.2	29.5	15.5	
70	70.6	95.8	90.5	84.7	78.1	70.6	59.5	47.0	33.3	18.0	
75	75.5	96.5	92.2	87.9	82.4	75.5	65.6	53.4	39.0	22.2	

SECOND-GROWTH BALSAM FIR (MEYER—1923)

Average	66.2	94.5	88.7	82.3	74.8	66.7	56.6	45.5	32.8	18.3	18

TABLE 4.—*Normal form quotients and adjusted percentile tapers*—Continued

BALSAM FIR—CULLED FOREST (BEHRE—1924)

Form class	Normal form quotient	Percentage of length from tip to breast height = x									Basis number of trees
		90	80	70	60	50	40	30	20	10	
		Adjusted percentile tapers									
Average	70.7	95.4	90.9	84.9	76.6	66.5	55.2	42.2	30.0	16.2	20
BALSAM FIR—OLD GROWTH (BEHRE—1924)											
55	59.5	93.6	85.6	77.4	67.7	55.5	44.1	34.7	24.3	12.0	11
60	62.9	93.9	87.4	79.8	70.8	60.3	49.6	38.4	27.4	14.8	21
65	69.2	95.2	90.4	84.1	75.4	64.6	53.2	40.8	29.8	17.3	18
70	71.4	95.6	90.8	86.0	78.9	69.7	59.0	47.6	33.6	20.5	18
75	77.6	96.8	92.7	89.0	84.2	75.9	64.7	51.4	37.1	21.6	5
LONGLEAF PINE (BROWN—1924)											
60	66.9	95.2	88.9	82.3	75.7	64.5	57.2	46.9	34.3	19.1	68
70	72.6	96.4	91.4	85.7	80.0	72.6	64.3	53.2	39.0	21.3	439
80	77.9	97.0	93.3	90.0	84.8	77.9	69.5	58.0	42.8	23.5	156
SHORTLEAF PINE (BROWN—1924)											
60	63.5	94.2	87.7	80.0	71.8	62.3	53.5	42.9	30.5	16.1	21
70	72.4	95.9	91.3	86.0	79.6	72.6	63.8	52.8	38.6	20.7	217
80	79.2	97.4	94.5	90.5	85.6	79.2	70.5	58.6	42.2	22.6	178
LOBLOLLY PINE (BROWN—1924)											
60	62.7	94.0	87.0	79.5	71.7	62.7	53.2	42.8	29.4	15.8	48
70	72.0	95.2	91.2	86.1	79.7	71.7	63.3	51.7	37.2	19.7	243
80	79.2	97.2	94.2	90.3	85.9	79.2	69.6	56.9	41.0	21.9	47
GRAY BIRCH (HAWLEY—1923)											
45	46.5	89.2	79.7	68.6	58.1	44.8	34.4	26.7	18.8	8.7	5
50	50.6	89.7	81.3	72.1	61.9	50.6	40.5	29.6	20.0	9.9	17
55	56.0	91.8	83.7	75.4	66.4	54.9	45.8	34.7	23.1	11.2	46
60	61.9	93.5	86.5	78.7	70.8	62.0	51.9	39.8	27.1	13.3	56
65	66.0	94.7	88.5	81.1	73.8	66.5	56.3	45.1	31.8	16.0	37
70	70.8	96.7	90.2	84.8	78.8	72.1	61.9	47.4	32.6	17.2	16

It should be borne in mind that the spread of the bands of points for the sections $x = 10$ and $x = 20$ would be materially reduced if series whose tops depart markedly from the equation $y = \frac{x}{a + bx}$, such as those for balsam fir, were plotted on a lower form-class line, for this would balance small diameters in the tops against larger diameters in the lower sections. This procedure, however, would increase the spread of the bands for the lower sections of the stem and make direct comparison of differences in the tops impossible. The spread of points indicated is materially lessened, especially in the zones for $x = 20$ and $x = 30$, when the data for balsam fir in Figure 12 are omitted. The spread of the points in these bands is indicative of the variations

between the different species. If the bands are narrow it means that there is little difference in the tapers of the different species, and that a single taper series may be used for all, but if the bands are broad and the points widely scattered within them considerable variation between the species is indicated, and the use of a generalized taper series for all the species becomes of doubtful accuracy.

CONSTANTS AND ABSOLUTE FORM FACTORS

To use the formula $y=\frac{x}{a+bx}$ as the basis for generalized taper tables requires the evaluation of the constants a and b. These constants vary with the form quotient and may readily be calculated by substituting the two pairs of values for the variables which are fixed for any given form class by definition of form quotient. Thus, for form class 75 we have $x=.50$, $y=.75$ and $x=1.00$, $y=1.00$.

Therefore,

$$.75=\frac{.50}{a+.50b} \qquad .667=a+.50b$$

$$1.00=\frac{1.00}{a+1.00b} \qquad 1.000=a+1.00b$$

Subtracting $\qquad .333=+.50b$

$$b=.667$$

Substituting $\qquad 1.00=a+.667$

$$a=.333$$

Table 5 gives the constants for form classes 40 to 90 at intervals of five units. Since in all instances $a+b=1$, it is only necessary to know one constant to solve the equation. In Figure 17 the constants are presented graphically in order to permit rapid approximation of the constants for any desired form quotient without calculation. With these constants percentile tapers for the entire stem can be computed, as presented in Table 5.

TABLE 5.—*Generalized percentile taper series; based on* $y=\frac{x}{a+bx}$

Form class	Percentage of length from tip to breast height									Constants		Absolute form factors
	90	80	70	60	50	40	30	20	10	a	b	
	Percentage of "normal" d. i. b.* at breast height											
40	85.71	72.73	60.87	50.00	40.00	30.77	22.22	14.29	6.90	1.5000	−0.5000	0.2688
45	88.03	76.60	65.62	55.10	45.00	35.29	25.96	16.98	8.33	1.2222	−.2222	.3008
50	90.00	80.00	70.00	60.00	50.00	40.00	30.00	20.00	10.00	1.0000	.0000	.3333
55	91.67	83.02	74.03	64.70	55.00	44.90	34.38	23.40	11.96	.8182	.1818	.3661
60	93.10	85.72	77.78	69.23	60.00	50.00	39.13	27.27	14.29	.6667	.3333	.4031
65	94.36	88.13	81.24	73.58	65.00	55.32	44.31	31.71	17.11	.5385	.4615	.4414
70	95.45	90.32	84.48	77.78	70.00	60.86	50.00	36.84	20.59	.4286	.5714	.4827
75	96.43	92.30	87.50	81.82	75.00	66.67	56.25	42.85	25.00	.3333	.6667	.5281
80	97.30	94.12	90.32	85.71	80.00	72.73	63.16	50.00	30.77	.2500	.7500	.5792
85	98.08	95.77	92.96	89.47	85.00	79.07	70.82	58.62	38.64	.1765	.8235	.6385
90	98.78	97.30	95.46	93.11	90.00	85.71	79.41	69.23	50.00	.1111	.8889	.7111

* D. i. b.=diameter inside bark.

The absolute form factor with the tree volume computed in 10 sections by the Smalian formula is expressed by the formula—

$$\frac{\frac{D^2}{2}+d^2_1+d^2_2+d^2_3+\cdots\cdots+d^2_9}{10D^2}$$

in which D, d_1, d_2, $d_3\cdots\cdots d_9$ are the diameters at breast height and at each succeeding tenth of the stem above breast height. This

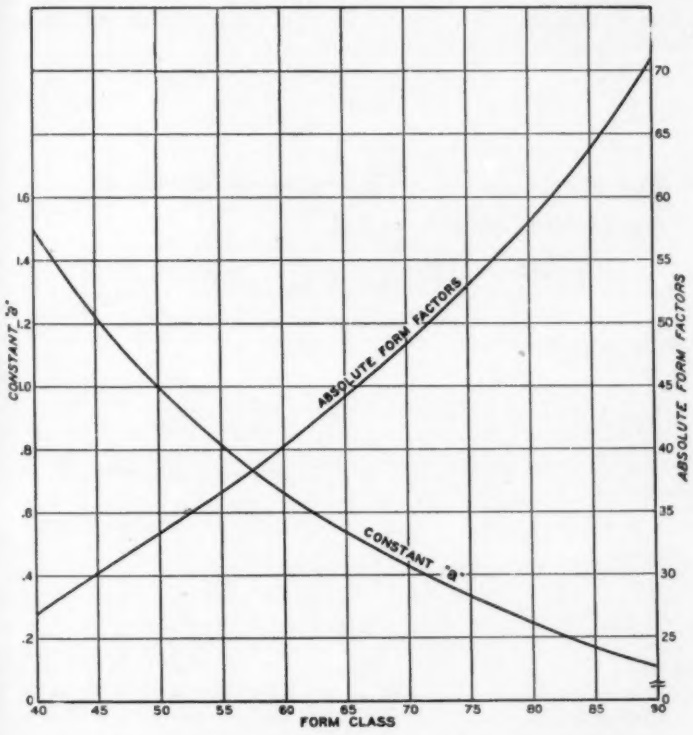

FIG. 17.—Absolute form factors and constants for $y=\frac{x}{a+bx}$. Constant $b=1-a$

method is in conformity with the conventional method of computing cubic contents, and was used by Jonson in compiling his volume tables. The Smalian formula, however, assumes the stem to be paraboloidal in form, and hence introduces a slight error which increases as the length of the sections into which the stem is divided increases. Absolute form factors corresponding exactly to the volumes generated by the stem curve, as expressed by the equation $y=\frac{x}{a+bx}$ revolving about the axis, can be computed with the aid of integral calculus.

The volume of a solid of revolution about the $x-$axis $=\pi\int_a^b y^2dx$.

The derivation of the equation of the stem curve has been worked in terms of diameters, but in computing the volume of the tree by the above integral y must be expressed as radius. Since the derived values of y are percentages of breast-height diameter they will apply as radii just as well as diameters. The limits of integration in our problem are from $x=0$, the tip of the tree, to $x=1$, breast-height point or unit height, and the result will be in terms of unit radius cubed. The cylinder with which the tree volume is to be compared will have radius 1, altitude 1 and volume of π in terms of unit radius cubed. The absolute form factor will then be—

$$\frac{\pi\int_0^1 y^2dx}{\pi}=\int_0^1 y^2dx,$$

or since $y^2=\frac{x^2}{(a+bx)^2}$, $\int_0^1\frac{x^2dx}{(a+bx)^2}=$ absolute form factor.

From a table of integrals we find

$$\int\frac{x^2dx}{(a+bx)^2}=\frac{1}{b^3}\left[a+bx-2a\log_e(a+bx)-\frac{a^2}{a+bx}\right]$$

Substituting $x=1$ gives

$$\frac{1}{b^3}\left[(a+b)-2a\log_e(a+b)-\frac{a^2}{a+b}\right]$$

Since we have found that $a+b-1$ in all cases this expression reduces to

$$\frac{1}{b^3}(1-a^2) \qquad \textbf{(A)}$$

Substituting $x=0$ gives

$$\frac{1}{b^3}\left(a-2a\log_e a-\frac{a^2}{a}\right)$$

or

$$-\frac{1}{b^3}(2a\log_e a) \qquad \textbf{(B)}$$

Subtracting B from A gives

Absolute form factor $=\int_0^1 y^2dx=\frac{1}{b^3}(1-a^2+2a\log_e a)$.

Absolute form factors derived in this way are included in Table 5, and are plotted in Figure 17 to permit rapid interpolation of the absolute form factor for any desired form quotient. If these absolute form factors are plotted on ratio or semilogarithmic paper they will fall very nearly into a straight line. This means that the form factors and hence the volumes in the portions above breast height are in a geometric progression and the rate of increase per unit increase of form quotient was found from the slope of the line to be approximately

1.9 per cent. The volume differences for the entire stems vary with total height and form quotient, since total volumes are related to the breast-high form factors, and so no general figure can be used with absolute accuracy. However, plotting breast-high form factors on semilogarithmic paper makes it possible to compute the approximate rate of increase of total volume for various ranges of heights and form quotients. It will be found that the rate of increase varies from less than 1 per cent for very short trees in the very low classes up to a little more than 1.6 per cent for the taller trees. A rate of 1.5 per cent represents a good average which will apply quite closely for all trees above 40 feet in total height except those most open-grown individuals. This means that the difference in volume corresponding to a form class interval of five units is about 7.5 per cent.

A method of approximate integration which appears to give slightly better results than that based on the Smalian formula is known as Simpson's rule. By this rule—

$$\int_0^1 y^2 dx = \frac{\Delta x}{3}\left(y^2_0 + 4y^2_1 + 2y^2_2 + 4y^2_3 + 2y^2_4 + \cdots + y^2_{10}\right)$$

By this rule the 10 sections into which the stem is divided are calculated in pairs according to Newton's prismoidal formula.

A direct expression of the effect of the variations of the different series from the tapers given by the formula may be obtained from the absolute form factors. The difference in absolute form factors between any two series may be converted into percentage difference in volume by dividing the difference in form factors by the form factor of the series used as a basis for comparison. Accordingly, the absolute form factors for the various series, excepting those for hemlock, and the corresponding volume error as compared to the formula, have been computed by the use of Simpson's rule for approximate integration and are tabulated in Table 6, columns 1 to 4. In this table the basis of comparison is the same as that used in Figure 16 (see pp. 698–701) except in a few series, notably those for balsam fir, where the falling off in the tops extends below the middle of the stem. What is wanted here as a basis for comparison is the form class into which the material would fall in practice, when corrected for root swell. Whenever the divergence of the tops from the formula extends below the middle of the stem, as indicated in Figures 11 and 12, the desired form class is represented by a line from breast height through the actual value for the mid-point, rather than by the lines in the figures, for these latter, in order to show directly the relative divergence in the tops of the various species, were drawn to fit the lower portions of the stem alone. When the actual form class of the material, with root swell eliminated, is taken for such series, the diameters in the lower portion of the stem are greater than those given by the formula and hence the volume errors may become positive in sign, even though relatively the series show a greater falling off in the tops than other series with negative errors.

70090—27——3

TABLE 6.—*Absolute form factors and volume errors*

WESTERN YELLOW PINE

Normal form class (1)	Absolute form factor: Formula figure 17 (2)	Absolute form factor: Material (3)	Volume error (4)	Basis number of trees (5)	Probable error normal variation (6)	Error above probable error (7)
			Per cent			
46.0	0.309	0.313	+1.3	3	1.6	----------
55.6	.371	.369	−.5	6	1.2	----------
60.2	.404	.405	+.2	17	.7	----------
66.1	.450	.448	−.4	29	.5	----------
70.2	.485	.486	+.2	55	.4	----------
75.0	.528	.527	−.2	70	.3	----------
79.1	.571	.560	−1.9	18	.7	−1.2
82.6	.610	.594	−2.6	3	1.6	−1.0

NORWAY SPRUCE (JONSON—1910)

62.2	0.419	0.420	+0.2	10	0.9	----------
70.7	.490	.491	+.2	30	.5	----------
77.6	.555	.557	+.4	7	1.1	----------

SCOTCH PINE (JONSON—1911)

65.6	0.447	0.447	----------	10	0.9	----------
71.9	.500	.498	−0.4	40	.4	----------
75.9	.538	.536	−.4	35	.5	----------
81.1	.592	.592	----------	9	.9	----------

EUROPEAN LARCH. HEIGHTS 8-14 M. (MATTSSON—1917)

61.8	0.417	0.418	+0.2	50	0.4	----------
63.9	.433	.430	−.7	37	.5	−0.2
66.8	.456	.454	−.4	52	.4	----------
67.9	.465	.464	−.2	54	.4	----------
70.8	.490	.490	----------	49	.4	----------
73.4	.514	.511	−.6	52	.4	−.2

EUROPEAN LARCH. HEIGHTS 15-22 M. (MATTSSON—1917)

64.1	0.435	0.437	+0.5	36	0.5	----------
66.7	.455	.454	−.2	42	.4	----------
70.5	.487	.487	----------	30	.5	----------
70.5	.487	.488	+.2	33	.5	----------
71.9	.500	.502	+.4	35	.5	----------
75.6	.535	.534	−.2	33	.5	----------

SIBERIAN LARCH (MATTSSON—1917)

55.4	0.369	0.369	----------	84	0.3	----------
61.1	.411	.410	−0.2	99	.3	----------
65.4	.445	.442	−.7	46	.4	−0.3

WESTERN WHITE PINE (BEHRE 1923. U. S. F. S. 1910-1913)

55.2	0.368	0.374	+1.6	4	1.4	+0.2
61.4	.413	.412	−.2	25	.6	----------
65.4	.445	.443	−.4	47	.4	----------
69.7	.481	.477	−.8	43	.4	−.4
73.7	.516	.515	−.2	16	.7	----------
80.2	.582	.576	−1.0	4	1.4	----------

TABLE 6.—*Absolute form factors and volume errors*—Continued

WESTERN WHITE PINE (CLAUGHTON-WALLIN AND McVICKER—1920)

Normal form class (1)	Absolute form factor: Formula figure 17 (2)	Absolute form factor: Material (3)	Volume error (4)	Basis number of trees (5)	Probable error normal variation (6)	Error above probable error (7)
71.2	0.494	0.500	*Per cent* +1.2	16	0.7	+0.5

DOUGLAS FIR (U. S. F. S. 1910–1913)

60.4	0.405	0.406	+0.2	5	1.3	----------
67.3	.460	.461	+.2	11	.9	----------
70.5	.487	.484	−.6	29	.5	−0.1
75.9	.538	.538	----------	17	.7	----------
80.6	.587	.570	−2.9	10	.9	−2.0

DOUGLAS FIR (CLAUGHTON-WALLIN AND McVICKERS—1920)

77.4	0.553	0.556	+0.5	11	0.9	----------

NORWAY PINE (CLAUGHTON-WALLIN AND McVICKER—1920)

65.8	0.448	0.444	−0.9	11	0.9	----------
70.8	.490	.485	−1.0	30	.5	−0.5
75.0	.529	.522	−1.3	40	.4	−.9

NORWAY PINE (REED—1924)

70.8	0.499	0.499	----------	21	0.6	----------
77.0	.549	.544	−0.9	15	.7	−0.2

YOUNG NORTHERN WHITE PINE (CLAUGHTON-WALLIN AND McVICKER—1920)

59.7	0.400	0.399	−0.2	5	1.3	----------
65.7	.447	.443	−.9	15	.7	−0.2
70.8	.490	.481	−1.8	9	.9	−.9

OLD NORTHERN WHITE PINE (WRIGHT—1923)

71.0	0.492	0.496	+0.8	342	----------	----------
70.6	.488	.489	+.2		----------	----------
71.5	.496	.491	−1.0		----------	----------
72.5	.505	.502	−.6		----------	----------
74.2	.521	.520	−.2		----------	----------
75.4	.533	.527	−1.1		----------	----------
76.4	.543	.536	−1.3		----------	----------
76.8	.547	.538	−1.6		----------	----------
80.2	.582	.564	−3.1		----------	----------

WHITE, RED, AND BLACK SPRUCE (WRIGHT, 1923)

61.1	0.411	0.412	+0.2	865	----------	----------
62.1	.418	.419	+.2		----------	----------
65.4	.445	.447	+.4		----------	----------
67.1	.459	.459	----------		----------	----------
70.1	.484	.480	−.8		----------	----------
72.3	.503	.496	−1.4		----------	----------
75.4	.532	.528	−.8		----------	----------
76.8	.547	.535	−2.2		----------	----------

TABLE 6.—*Absolute form factors and volume errors*—Continued

WHITE SPRUCE (CLAUGHTON-WALLIN AND McVICKER, 1920)

Normal form class (1)	Absolute form factor: Formula figure 17 (2)	Absolute form factor: Material (3)	Volume error (4)	Basis number of trees (5)	Probable error normal variation (6)	Error above probable error (7)
			Per cent			
65.9	0.449	0.450	+0.2	6	1.2	----------

VIRGIN RED SPRUCE—MAINE (U. S. F. S. 1902)

52.4	0.349	0.359	+2.9	7	1.1	+1.8
62.0	.418	.417	−.2	29	.5	----------
69.8	.482	.477	−1.0	65	.4	−.6
77.6	.555	.541	−2.5	31	.5	−2.0

SECOND GROWTH RED SPRUCE—NEW HAMPSHIRE (U. S. F. S. 1903)

72.6	0.506	0.499	−1.4	20	0.6	−0.8

RED SPRUCE CULLED FOREST—NEW HAMPSHIRE (U. S. F. S. 1903)

60.8	0.409	0.416	+1.7	6	1.2	+0.5
71.4	.495	.484	−2.2	30	.5	−1.7
78.5	.564	.541	−4.1	8	1.0	−3.1

RED SPRUCE OLD FIELDS—NEW HAMPSHIRE (U. S. F. S.—1903 AND MEYER, 1924)

55.3	0.368	0.385	+4.6	3	1.6	+3.0
60.3	.405	.406	+.2	4	1.4	----------
65.2	.444	.446	+.5	22	.6	----------
69.9	.483	.479	−.8	39	.5	−.3
74.9	.527	.514	−2.5	17	.7	−1.8
78.9	.567	.542	−4.4	7	1.1	−3.3

BALSAM FIR (WRIGHT—1923)

60.0	0.403	0.407	+1.0	362	----------	----------
65.0	.441	.439	−.5		----------	----------
70.6	.488	.478	−2.0		----------	----------
75.5	.534	.522	−2.2		----------	----------

BALSAM FIR, SECOND GROWTH—MAINE (MEYER, 1923)

66.2	0.451	0.452	+0.2	18	0.7	----------

BALSAM FIR, CULLED FOREST—MAINE (BEHRE-1924)

66.5	0.454	0.458	+0.9	20	0.6	+0.3

BALSAM FIR, OLD GROWTH—MAINE (BEHRE-1924)

55.5	0.370	0.385	+4.1	11	0.9	+3.1
60.3	.405	.413	+2.0	21	.6	+1.4
64.6	.438	.447	+2.1	18	.7	+1.3
69.4	.478	.482	+.8	18	.7	+.1
75.9	.538	.524	−2.6	5	1.3	−1.3

TABLE 6.—*Absolute form factors and volume errors*—Continued

LONGLEAF PINE (BROWN-1924)

Normal form class	Absolute form factor		Volume error	Basis number of trees	Probable error normal variation	Error above probable error
	Formula figure 17	Material				
(1)	(2)	(3)	(4)	(5)	(6)	(7)
			Per cent			
66.9	0.457	0.455	−0.4	68	0.3	−0.1
72.6	.506	.505	−.2	439	.1	−.1
77.9	.558	.550	−1.4	156	.2	−1.2

SHORTLEAF PINE (BROWN-1924)

63.5	0.430	0.428	−0.5	21	0.6	----------
72.4	.504	.503	−.2	217	.2	----------
79.2	.572	.559	−2.3	178	.2	−2.1

LOBLOLLY PINE (BROWN-1924)

62.7	0.423	0.425	+0.5	48	0.4	+0.2
72.0	.501	.496	−1.0	243	.2	−.8
79.2	.572	.553	−3.3	47	.4	−2.9

GRAY BIRCH (HAWLEY-1923)

46.5	0.312	0.314	+0.6	5	1.3	----------
50.6	.338	.340	+.6	17	.7	----------
56.0	.373	.373	----------	46	.4	----------
61.9	.417	.413	−1.0	56	.4	−0.6
66.0	.449	.446	−.7	37	.5	−.2
70.8	.490	.485	−1.0	16	.7	−.3

The figures in column 4, Table 6, indicate the net volume errors of each particular series as compared to the formula. An examination of these figures shows a fairly consistent tendency for the errors to increase from the lower to the higher form classes. This is a measure of the effect of the divergence from the straight lines noted in Figures 2 to 14. But these figures afford no basis for an estimation of how much of these errors are really the result of divergence from the formula and how much is simply the result of the normal variations of trees included in a given form class average which will be influenced by the number of trees in the average. Furthermore, they give no measure of the variation of the diameters at any point on the stem as compared to the variations of the mid-points which are arbitrarily limited to the form-class interval.

To determine the importance of the error from these sources an analysis was made of the western yellow and western white pine material, since these species agree very closely with the formula. The diameter at each tenth of the stem of each tree was compared to the diameter given by the formula for a form quotient corresponding to the actual average of the class to which the tree belonged. A given deviation at the base of a tree will have a much greater influence upon the volume than a similar deviation in the upper portions of the stem, and a given deviation in a tree of low form quotient will have a relatively greater weight than in a tree of higher

form quotient because the basis of comparison is relatively smaller. To make the observed deviations comparable they should be expressed in terms of their effect upon volume independent of their position in the tree, or the size or form of the tree. Since the diameters have been expressed on a percentile basis, the effect of size of the trees has already been taken care of. To express the effect upon volume of a given percentile deviation in diameter, independent of its position on the stem, a general expression for the resultant rate of change in volume per unit length can be used. The volume of the tree may be considered as made up of a number of cylinders of varying diameters corresponding to the diameters at different points on the stem. As the altitude of these cylinders approaches zero the sum of their volumes approaches the true volume of the tree. An increase or decrease of diameter at any point on the stem may therefore be considered as affecting a cylinder having a diameter equal to that of the tree at the given point and an altitude equal to the interval between successive measurements, in this case one-tenth of the total height.

Let V = volume
Y = diameter at any point on the stem
y = deviation of diameter from formula values
H = height or altitude
n = number of trees
f = absolute form factor
AD = average deviation
σ = standard deviation

Then, for a cylinder

$V = \frac{\pi}{4} Y^2 H$, and by differential calculus

$\frac{dV}{dY} = \frac{\pi}{2} HY$ = rate of change of volume with change of diameter

Since $H = \frac{1}{10}$,

$y\frac{\pi}{20} Y$ = change in volume per tenth of length resulting from deviation, y, in diameter.

$\frac{\pi}{20} AD_y \ Y$ = variability of changes in volume per tenth of length corresponding to deviations in diameter = average error in terms of volume per tenth of length in estimate of diameter at any point by the equation of the stem curve.

$V = \frac{\pi}{4} Y^2 Hf$ = volume of tree.

Since total height = 1, D. B. H. = 1, and since the standard deviation is approximately 1.253 times the average deviation,

$$\sigma \text{ per } \tfrac{1}{10} \text{ length in percentage of volume} = \frac{\frac{\pi}{20} AD_y \ Y}{\frac{\pi}{4} f} \cdot 1.253 \cdot 100$$

$$= \frac{25.06}{f} AD_y \ Y.$$

This last expression gives the standard error in percentage of volume resulting from deviations in diameter at any one tenth of the stem.

In order to compare the variations in diameters at different points on the stem with variations at the mid-point, which are limited by the form-class interval, the standard deviations in volume at each tenth of the stem in the different form classes are shown in Table 7 and Figure 18 for the western yellow pine and western white pine material, these species being selected because they fit the formula well.

TABLE 7.—*Relative effect of variability of upper diameters upon volume estimates by formula, shown in percentage of volume*

WESTERN YELLOW PINE

Form class	Percentage of length from tip to breast height									Number of trees
	90	80	70	60	50	40	30	20	10	
	Standard errors in percentage of volume									
45	0.743	1.029	1.032	1.456	0.164	0.965	0.345	0.185	0.077	3
55	.464	1.849	.582	.829	.308	.343	.571	.249	.098	6
60	1.004	1.041	.986	.800	.511	.642	.658	.245	.157	17
65	.763	.803	.690	1.118	.390	.664	.688	.539	.215	29
70	.831	.958	.974	.794	.402	.859	.740	.652	.326	55
75	.652	.816	.980	.779	.416	.636	.710	.714	.417	70
80	.440	.687	.851	.649	.345	.716	1.129	1.220	1.598	18
85	.308	.549	.265	.714	.240	.740	2.064	2.486	.954	3
Average and total	.723	.890	.927	.832	.407	.714	.757	.685	.466	* 198

WESTERN WHITE PINE

60	0.756	0.682	0.880	0.818	0.419	0.567	0.575	0.367	0.207	23
65	.731	.889	.794	.794	.428	.726	.749	.580	.307	47
70	.646	.771	.771	.562	.422	.729	.750	.734	.299	43
75	.392	.690	.608	.498	.313	.554	.651	.802	.486	16
Average and total	.661	.790	.776	.678	.410	.680	.711	.634	.315	129

* Three trees in form class 85 are omitted from the average because this class does not fit the formula in the tops.

From these figures it appears that the variations of the diameters at points between the mid-point and the base or the tip involve volume errors seldom over twice the errors arising from the spread of the form-class interval itself. Furthermore, in series which fit the formula well, deviations in diameters in the lower half of the stem are more important in their effect on volume than those in the upper portion, although the latter are high in absolute values. The extreme similarity of the results for the two species also indicates that variation in this respect is probably identical for all practical purposes in all species, and that an evaluation of the total volume error in estimating upper diameters from the formula may be applied quite generally.

If the deviations at the various points on the stem are uncorrelated, the total error resulting from simultaneous deviations at all points will be given by the formula for the standard deviation of a sum

$$\sigma_s = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \cdots}$$

It is obvious, however, that deviations at different sections are often correlated with each other. Trees in conformity with the formula but falling near either the upper or lower limits of the form-class interval will have very high positive correlation between deviations at any pair of sections. On the other hand many trees with positive deviations in the lower portions of their stem tend to have negative errors in their tops and vice versa. In such cases, the correlation between deviations in adjacent section in either upper or lower portions of the stem will be positive, but between a deviation from the upper portion and one from the lower portion the correlation will be negative. In

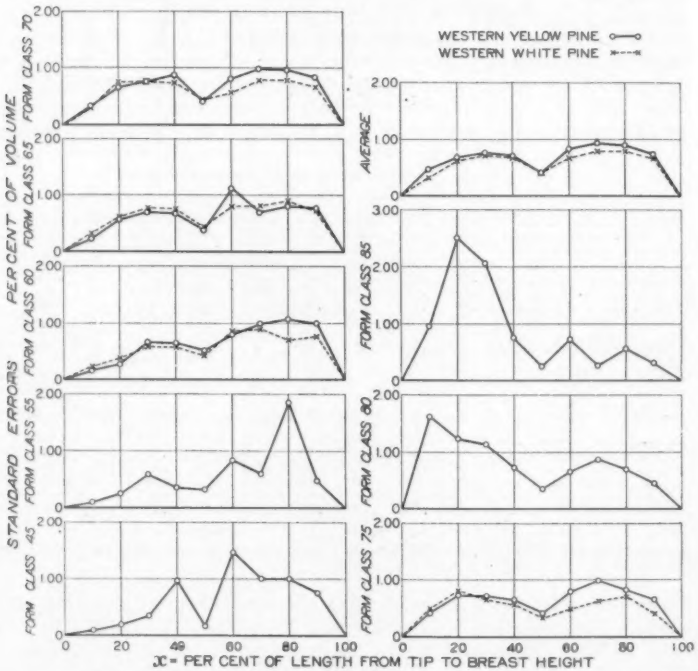

FIG. 18.—Relative effect of variability of upper diameters upon volume estimates by formula

addition there will be many trees which exhibit no correlation at all between deviations at different sections.

To make the formula for the standard deviation of a sum as given above applicable in the case of correlated deviation requires the addition under the radical of as many terms of the form $2r_{12}\,\sigma_1\sigma_2$ (in which r_{12} is the coefficient of correlation between deviations at any two sections) as there are pairs of sections. Since there will be 36 such terms to cover all the possible combinations of 9 sections taken in pairs, this involves a volume of work out of all proportion to the value of the result sought. We are justified therefore in using the simple formula for uncorrelated deviations as the basis for comparison, with an approximate correction for correlation of deviations

applied later to the final average figures. The total standard error in percentage of volume resulting from estimation of all upper diameters from the formula will therefore be approximately:

$$\sigma_s = \sqrt{\Sigma\sigma^2} = \sqrt{\Sigma\left\{\frac{25.06\ AD_y.\ Y}{f}\right\}^2}$$

To estimate the magnitude of the correction which it is necessary to apply to the results from the approximate formula, to account for correlation of deviations between different sections, average values may be assigned to the factors of the 36 terms of the form $2r_{12}\ \sigma_1\ \sigma_2$. The average product of the standard volume error caused by the individual deviations at the different sections ($\sigma_1\ \sigma_2$) may be taken as the square of the average of the errors at the different sections, or $\left(\frac{\Sigma\sigma}{9}\right)^2$.

For an approximation of the average value of r we may assume that any pair of deviations with like sign indicates positive correlation, that any pair of deviations with opposite signs indicates negative correlation, and that the tendency toward negative correlation would offset the tendency toward positive correlation.

This is in effect exactly parallel to the method of expressing correlation by the approximate method of concurrent deviations given by King (*16*) as—

$$r = \pm\sqrt{\pm\frac{2c-u}{u}}$$

In this formula r = the coefficient of correlation, c = number of pairs of deviations with like sign, and u = total number of pairs in the sample. The ± signs both before and under the radical are used to indicate that when the quantity $\frac{2c-u}{u}$ is negative the coefficient is negative, and when it is positive the coefficient is positive.

To determine the number of pairs of deviations with opposite signs in a given sample, tally each tree according to the number of deviations of positive sign which it exhibits in its nine sections. If n_+ and n_- represent the number of deviations with positive and negative signs, respectively, in any tree, the number of pairs with unlike sign which may be formed will be given by the formula $C_{+-} = n_+\ n_-$, since n_+ positive deviation may be selected and each selection matched with n_- negative deviation. The total number of pairs of deviations which may be taken from nine sections is given by the formula $C_{9\ .\ 2} = \frac{9!}{2!\ (9-2)\ !}$. This has already been given as 36. From the tally of trees according to number of deviations of positive sign and the corresponding numbers of pairs with opposite signs which may be formed, the total number of pairs with opposite sign is obtained. By subtracting this number from the total number of pairs in the sample the number of pairs with like signs is obtained. Substitution in King's formula will then give the value of r, the net proportion of full correlation. The procedure is illustrated by random samples of 100 western yellow pines and 106 western white pines in Table 8. The net proportion of full positive correlation

exhibited by the western yellow pine is 0.415 and by the western white pine 0.425. It is thus evident that an average value for r would be somewhere between these figures and that 0.42 would doubtless be a sufficiently conservative figure.

TABLE 8.—*Approximation of average correlation between deviations in diameter at different sections*

Number of sections with +deviation	Number of pairs with opposite sign	Western yellow pine [a]		Western white pine [b]	
		Number of trees	Number of pairs with opposite sign	Number of trees	Number of pairs with opposite sign
9	0	5	0	11	0
8	8	8	64	8	64
7	14	8	112	8	112
6	18	18	324	18	324
5	20	14	280	15	300
4	20	13	260	17	340
3	18	13	234	14	252
2	14	12	168	11	154
1	8	6	48	2	16
0	0	3	0	2	0
----------	----------	100	1,490	106	1,562

[a] Total number of pairs (u) = 100×36=3,600. Number of pairs with like sign (c) = 3,600−1,490=2,110. $r=\pm\sqrt{\frac{\pm 2c-u}{u}}=+0.415$.

[b] Total number of pairs (u) = 106×36=3,816. Number of pairs with like sign (c) = 3,816−1,562=2,254. $r=+0.425$.

From these figures the correction of the formula for the standard deviation of the sum may be approximated as follows:

$$36(2r_{12}\sigma_1\sigma_2)=36\cdot 2\cdot 0.42\left(\frac{\Sigma\sigma}{9}\right)^2=.373(\Sigma\sigma)^2$$

The standard volume error for the entire stem resulting from deviations of upper diameters from the equation of the stem curve, corrected for correlation between deviations at different sections, may then be stated as

$$\sigma_s=\sqrt{\Sigma\sigma^2+.373(\Sigma\sigma)^2} \text{ or}$$

since σ has already been shown to be $25.06\,\frac{AD_y\cdot Y}{f}$

$$\sigma_s=\sqrt{\Sigma\left(\frac{25.06\,AD_y\cdot Y}{f}\right)^2+.373\left(\frac{25.06}{f}\Sigma(AD_y\cdot Y)\right)^2}$$

The standard errors for entire trees computed separately for each form class by the formula $\sigma_s=\sqrt{\Sigma\sigma^2+.373\ (\Sigma\sigma)^2}$ from the data of Table 7 are shown in Table 9 and Figure 19 for western yellow pine and western white pine. The standard error appears to be almost constant except in the highest form classes of the western yellow pine, where the divergence of the tops from the formula increases the error considerably above what would be expected from the errors

within series conforming more exactly to the formula. In making the calculation for western white pine the data for form classes 55 and 80, four trees in each class, were omitted in order to restrict the calculation to series agreeing very closely with the formula as shown by Table 6. For the same reason the three trees of form class 85 in western yellow pine were omitted from the calculation of the average standard error.

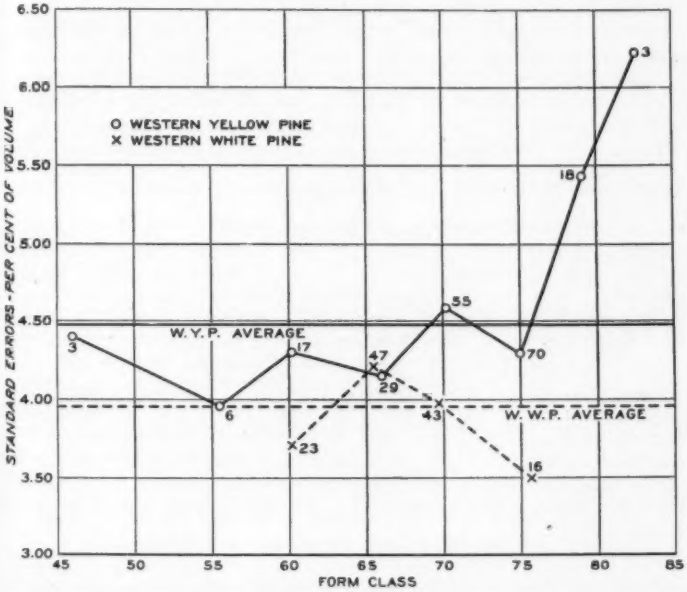

FIG. 19.—Standard errors in volume caused by deviations of upper diameters from the formula

TABLE 9.—*Approximate standard errors in percentage of volume caused by deviations of upper diameters from the formula*

Form class	Western yellow pine			Form class	Western white pine		
	Actual average form quotient	Standard error volume	Number of trees		Actual average form quotient	Standard error volume	Number of trees
		Per cent				*Per cent*	
45	46.0	4.40	3	60	60.1	3.72	23
55	55.6	3.96	6	65	65.4	4.21	47
60	60.2	4.30	17	70	69.7	3.98	43
65	66.1	4.15	29	75	73.7	3.50	16
70	70.2	4.59	55				
75	75.0	4.29	70				
80	79.1	5.43	18				
85	82.6	6.22	3				
Average	70.44	4.48	198	Average	66.92	3.96	129

For the entire 198 western yellow pines with average form quotient of 70.44, the standard error caused by deviations in upper diameters was 4.48 per cent and for 129 western white pines averaging 66.92 in form quotient the standard error from this source was 3.96 per cent. The difference between the figures obtained for the two species may be due in part to the fact that western white pine is in general a smoother, more regularly shaped tree than western yellow pine, but the difference is so slight that it will be safe to assume an average figure for general application.

Taking an average of these figures, say 4.2 per cent, as the standard volume error due to the normal variation of upper diameters of the trees included in any form class, the net volume errors of the different series given in column 4, Table 6, can be modified to determine whether or not the errors indicate significant divergence from the formula. The probable error of an average is given by the formula $PE_a = \frac{.6745\sigma}{\sqrt{n}}$ in which σ is the standard deviation of the individual trees, 4.2 per cent this case, and n is the number of trees in the average. Column 6, Table 6, calculated from this formula, gives the probable error in each series attributable to the normal variation of upper diameters. Any volume error exceeding this probable error may be considered as indicative of significant divergence from the formula, since the chances are more than equal that such an error would arise from normal variation of the upper diameters rather than from the sampling alone. On this basis column 7, Table 6, shows the net volume error attributable to divergence from the formula in the different series. It will be noted that for about half the series listed the errors of column 4 are of a magnitude which might result from simple sampling alone, and that errors attributable to divergence from the formula seldom exceed ± 1 per cent except in the large form classes.

ANALYSIS OF VARIATIONS FROM THE FORMULA

In the material presented above a definite and constant tendency was noted in many of the species for the tops in the larger form classes to fall below the values given by the formula. With eastern hemlock the form of the entire tree differs markedly from that given by the formula, but the divergence appears to be an exaggerated case of the type noted for the other species. An attempt has been made to analyze the nature of these variations in a search for a more perfect expression of stem form.

A generalized taper series should give values representing an average for all the species to which it applies, and hence should give lines through the centers of the bands of points in Figure 16, which indicate the spread of values for the various species at each tenth of the stem.

The extremely limited range of the belts of points in the chart and the similarity of form factors as shown above leave little doubt but that all the species studied except hemlock may be measured with a single set of taper curves. The lines representing the formula $y = \frac{x}{a + bx}$, however, fall near the upper limits of the belts of points in the top sections of the larger form classes rather than through the centers, and the question therefore arises as to the possibility of

finding an equation more nearly representing the average in all cases without sacrificing the simplicity of the formula already deduced.

By drawing a smooth curve through the center of each belt of points, an average taper series may be obtained. This series may be harmonized by replotting, and used directly as the basis for volume and taper tables, but, inasmuch as the original placing of the curves and their subsequent harmonization would vary with the individual doing the work as well as with the material used as a basis, such a series could never have the nicety and precision of a series based on a mathematical formula nor lend itself so well to analysis and comparison.

Considerable study was, therefore, devoted to the problem of fitting an equation to the average taper series obtained from the chart, but none was found which seemed superior to $y=\frac{x}{a+bx}$ from the standpoint of simplicity and exactness as well as closeness of fit.

From this study it is quite apparent that any improvement over the equation $y=\frac{x}{a+bx}$ must involve more than two constants. Inasmuch as the definition of form quotient fixes only two points on the curve, the evaluation of constants in an equation with more than two constants necessitates resort to a method of approximation, and the resulting constants will vary according to the empirical values used in the calculation. Furthermore, in problems such as this, where separate constants must be derived for each form class, the constants should progress systematically from one form class to another. To accomplish this requires plotting the constants derived from the data in order to smooth out the irregularities between form classes. The constants derived by this process are, therefore, subject to variation, and as a result the equation loses much of its value as an exact and precise measure of the stem curve. It is no longer entirely independent of the errors of sampling in the field data nor of personal judgment in manipulation. It does not seem that the slight improvement in closeness of fit and reduction of volume errors in the larger form classes which may be obtained by an equation with more than two constants is sufficient to warrant the sacrifice of the simplicity and exactness of the equation $y=\frac{x}{a+bx}$.

It is of interest, however, in support of the general theory that all trees taper in accordance with the same law and that their taper may therefore be expressed by a single type equation, to study the fit of more complex equations to the average series and their adaptability to the peculiarities of the individual species. In this investigation a modification of the original equation, such as $x/y=a+bx+c \log x$ or $x/y=a+bx+ce^{dx}$, seems to give the most satisfactory fit under all conditions.

Between these two equations there seems to be little choice from the standpoint of ability to fit closely any of the individual series. For some species the former type is slightly superior and for others the latter more truly represents the shape of the curves. In attempting to evaluate the constants to fit the average series derived graphically it was found exceedingly difficult if not impossible to harmonize the constants from one form class to the next with the use of the

equation $x/y=a+bx+ce^{dx}$, but little difficulty was encountered in deriving a satisfactory set of constants for the equation $x/y=a+bx+c \log x$. For either type equation, however, constants may apparently be found which will give values exceedingly close to any of the individual taper series examined, including hemlock. This seems to indicate that the minor variations noted between different species are all of the same general type and that all may be expressed in terms of the same mechanical law.

Using the constants given in Table 10, which were derived from the graphic average series as outlined above, taper series have been computed for the equation $x/y=a+bx+c \log x$ and the results are shown in Figure 16 for comparison with the original equation and the material.

TABLE 10.—*Constants for* $x/y=a+bx+c \log x$

Form class	a	b	c	Form class	a	b	c
45	1.222	−0.222	0.000	70	0.515	0.608	−0.0615
50	1.000	.000	.000	75	.448	.716	−.0820
55	.822	.186	−.0030	80	.393	.812	−.1025
60	.696	.346	−.0210	85	.347	.898	−.1225
65	.596	.486	−.0410				

To show how slight is the effect on volume computations of the refinements gained by this series, the absolute form factors have been computed by Simpson's rule for approximate integration and are given in Table 11 together with those derived from the equation $y=\frac{x}{a+bx}$. The constant c has no appreciable value until form class 60 is reached, so that up to this point there are no differences in the absolute form factors.

TABLE 11.—*Comparison of absolute form factors*

Form class	Absolute form factors		Difference	Volume difference
	$x/y=a+bx+c \log x$	$y=\frac{x}{a+bx}$		
				Per cent
60	0.4032	0.4031	+0.0001	+0.02
65	.4413	.4414	−.0001	−.02
70	.4816	.4827	−.0011	−.23
75	.5257	.5281	−.0024	−.46
80	.5742	.5792	−.0050	−.86
85	.6276	.6385	−.0109	−1.71

It will be noted that up to form class 75, which is seldom if ever exceeded as an average in forest stands, there is not over 0.5 per cent difference in the total cubic volumes as measured by the two equations and, as the tapers vary most widely in the tips, a large part of this small volume difference lies in the unmerchantable portions of the stem.

EFFECT OF VARIOUS FACTORS UPON THE FORM CURVE

In the outline of the development of the form-class system in Sweden it was pointed out that previous to 1911 European foresters had established the fact that taper based on form quotient was

independent of diameter, height, locality, and, to a certain extent, species. On the other hand Wright (*26*), after considerable work with Canadian species, states that taper within a given form class varies with species, diameter, age, and probably height also. Baker (*1*) and Behre (*4*) have both shown that the differences found by Wright practically disappeared when the distortion of the breast-high-diameter by root swell was eliminated from the material. The wide variety in the character of the material included in the present study tends to confirm the European conclusions in this respect, yet it is worth while to inquire more in detail into the effect of various factors upon the form curve in order to check up on this point.

The western yellow pine and red spruce data are especially suitable for this analysis because they cover such a wide range of conditions. With the aid of mechanical sorting and tabulating machines, these

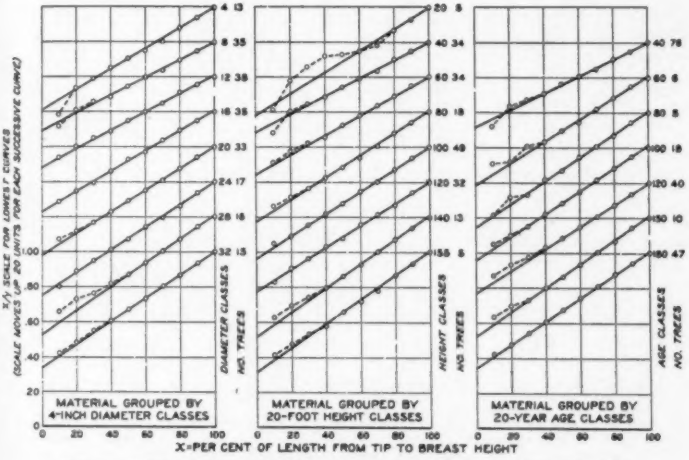
MATERIAL GROUPED BY 4-INCH DIAMETER CLASSES
MATERIAL GROUPED BY 20-FOOT HEIGHT CLASSES
MATERIAL GROUPED BY 20-YEAR AGE CLASSES
DIAMETER CLASSES NO. TREES
HEIGHT CLASSES NO. TREES
AGE CLASSES NO. TREES
X=PER CENT OF LENGTH FROM TIP TO BREAST HEIGHT

FIG. 20.—Effect of diameter, height, and age on form curve of western yellow pine

data were therefore sorted and averaged separately for each of the factors for which notes or measurements were available.

The method of analysis used for each factor is illustrated by the following instance of grouping by diameter classes. All the trees of a given species were first sorted into 4-inch d. b. h. classes. The percentile tapers were then added together for each 4-inch class to get an average taper series for each class regardless of form quotient. For the series thus obtained corresponding values of x/y were computed, and these were then plotted to note their conformity to the formula and for comparison with one another. If the form curve of the species varies with diameter the smaller trees will exhibit in this plotting a different tendency from that of the larger trees. Comparison of the tendency exhibited by these series with those evident in grouping by other factors gives further indication of the effect of the various factors upon the form curve.

Figure 20 shows the result of averaging the western yellow pine material by diameter, height, and age. No important or consistent

divergence from the straight lines are discernible in any of the groupings, so that we may conclude that in general the conformity of western yellow pine to the formula is independent of diameter, height, and age.

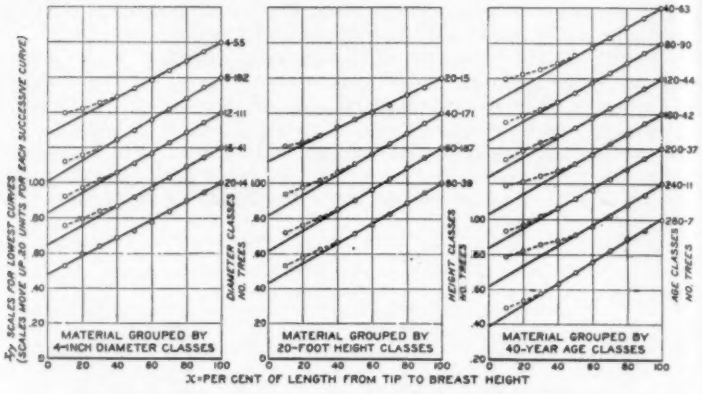

FIG. 21.—Effect of diameter, height, and age on form curve of red spruce

Figures 21 and 22 show the result of averaging the red spruce data by diameter, height, age, crown class, forest type, and site. In practically all the series this species shows a divergence of the tips below the formula but the differences in the amount and character of this divergence are very slight in most instances.

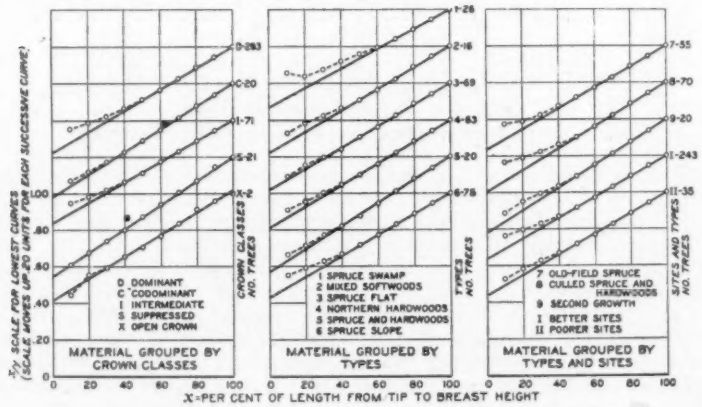

FIG. 22.—Effect of crown class, forest type, and site on form curve of red spruce

From the grouping by diameter classes it appears that the larger sizes hold their taper somewhat better in the tips than the small and average size classes.

The type of form curve appears in no way affected by height or age.

In the grouping by crown classes the suppressed trees hold their taper in the tips much better than the other classes and the greatest divergence of the tops from the formula occurs in the dominant class. The fact that the open-grown trees do not show any divergence of the tops from the formula as might be expected in comparison with the dominant class is hardly significant in view of the small number of trees upon which the figures are based.

In the grouping by forest types the terminology is in accordance with the definitions of forest types adopted by the New England Section of the Society of American Foresters (*9*) except for Nos. 8 and 9. Type 8, "culled spruce and hardwoods," was differentiated from type 5, "spruce and hardwoods," in order to bring out any difference in form which may result from the changed conditions under which the residual stand develops after a heavy culling. Type 9, "second growth," was differentiated to cover a number of trees so designated on the original tally sheets, which probably represent second-growth stands resulting from fire, windfall, or other natural phenomena, differing radically in character from the usual old growth of the various types but still not partaking of the characteristics of the old-field type.

The divergence of the tips below the formula is most pronounced in the culled spruce and hardwoods type, and considerably more than the average in old-field and spruce-swamp types. But none of these variations are of much importance, nor are they in excess of similar divergencies appearing at random in some of the other groupings. It may be noted, however, that figures from western white pine left after selective logging 15 years previously showed a very similar tendency to differ slightly in form from trees grown in undisturbed forest conditions throughout their existence. This is doubtless the result of an entirely different distribution on the stem of the wood produced each year, caused by the sudden change in exposure following opening up of the stand. It is also reasonable to suppose that as the trees develop further they will again adjust themselves to the new conditions in accordance with the mechanical laws which control the distribution of wood, and eventually regain a form curve similar to that found in natural and undisturbed stands.

On better sites the divergence of the tops below the formula appears greater than on poorer sites, but again the difference is not important.

From this summary we may conclude that, although minor tendencies to differ in form type may be observed, none of them are of sufficient importance to warrant special treatment; and accordingly for all practical purposes we may consider tapers based on the formula to apply equally well under all conditions. It should be noted, however, that these analyses apply only to the type of form curve and that nothing is indicated here as to the effect of the various factors upon the average form quotient or upon the thickness of bark and amount of root swell at the breast-height point. These matters are the subject of further investigations now under way.

DEVELOPMENT OF TOTAL CUBIC FOOT VOLUME TABLES

Total cubic foot volume tables may be derived from the absolute form factors in Table 5 by the method used by Jonson (*13*) and outlined in Chapman's Forest Mensuration (*6*), which involves the

derivation of breast-high form factors as the basis for computation of cubic contents.

In this procedure the normal diameter at the ground is calculated by prolonging the equation of the stem curve down to the ground separately for each height class. In the same way a new middle

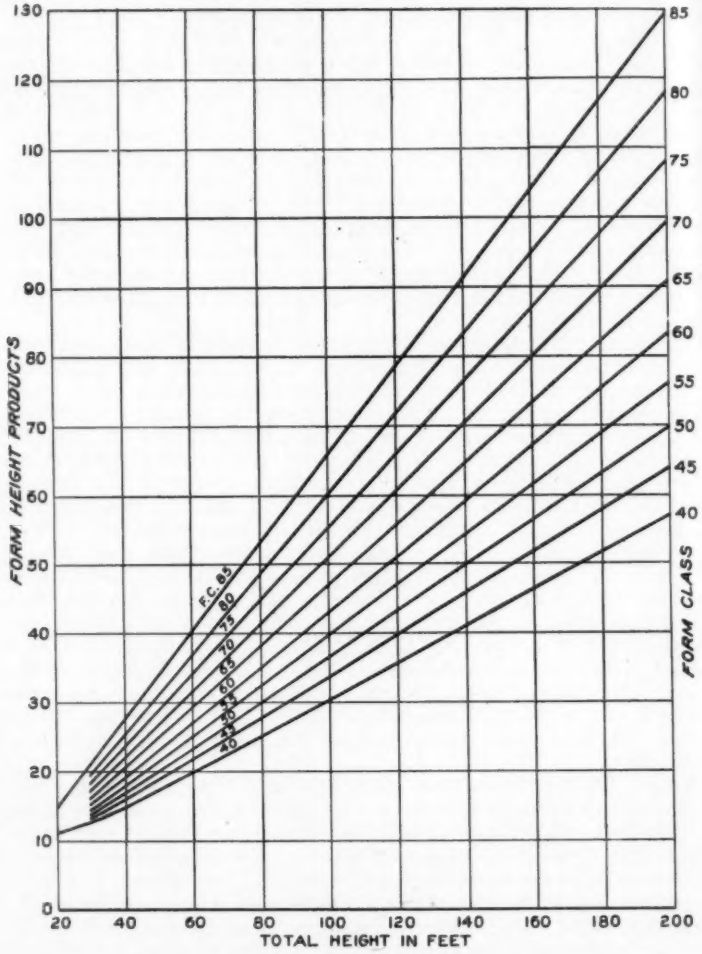

FIG. 23.—Form-height products based on $y=\frac{x}{a+bx}$ for total cubic contents inside bark

diameter is computed for each height class on the basis of total height. These values give a new set of absolute form quotients for which corresponding absolute form factors are interpolated from the curve of Figure 17. These absolute form factors are then converted into breast-high form factors.

From a set of breast-high form factors computed in this way form-height products were derived by multiplying each factor by its corresponding height. These form-height products are shown in Figure 23. They are based on total cubic foot contents of the stem without bark. The chart permits interpolation of the form height products for trees of any height from 20 to 200 feet and for any form class from 40 to 85. In the construction of this chart calculations of the form height products were made only for height classes 20, 25, 30, 40, 60, 80, 120, 160, and 200 in form classes of 5 units each.

The next step is to multiply the form-height products by the basal areas corresponding to the different inch classes for the range likely to be needed in any particular case. The resulting volume tables will show the contents of trees according to their normal diameter inside bark, and will serve for all species which conform approximately to the type equation $y=\frac{x}{a+bx}$. This step can be accomplished most economically by plotting the tables on logarithmic coordinate paper. Since the plotting of volume on diameter on logarithmic paper gives a series of straight lines for different height classes, the lines can be accurately placed from the computation for only two or three selected diameter classes.

The final step in the completion of the tables must be taken separately for each species (or group of species) because of differences in bark thickness and amount of root swell at breast height; for practical reasons the final volume tables should be based on the usual breast-height measurement rather than normal diameter inside bark. The normal diameters corresponding to the different d. b. h. classes must, therefore, be determined separately for each species. With this relation worked out, as will be illustrated later, the transformation to the final volume tables can best be done on the logarithmic plotting of the tables for each form class, cubic foot contents on normal diameters with a curve for each height class, by drawing vertical lines on the abscissae corresponding to even inches of d. b. h.

TAPER CHARTS FOR DIAMETER, HEIGHT, AND FORM CLASSES

As a basis for volume tables in board feet, ties, poles, or other units of product it is necessary to convert the percentile taper series of Table 5, derived from the generalized equation, into taper tables or charts giving diameters and heights in absolute rather than relative terms. To work out arithmetically from the percentile taper series the diameters at specified heights for all sizes and form classes required in a volume table is quite tedious and serves only the purpose of that particular table. At any time when it may be desired to make a new table using different standards of utilization or based on a different class of products the entire job may have to be repeated.

It is, therefore, not only desirable but economical to put the taper tables into permanent form for general use with all species to which the basic equation of the stem curve applies. This can be done to best advantage by a graphic method suggested by Hohenadl (*12*).

The percentile tapers from Table 5 are first plotted on logarithmic coordinate paper and smooth curves drawn through the points for

each form class. These curves are then transferred to a sheet of transparent tracing paper. The actual diameters in inches at any height in feet for trees of any size can now be read directly by shifting the curves on the respective axes for diameters and heights. This procedure is illustrated by Figure 24, in which only form class 70 is shown. Curve I shows the preliminary plotting of the percentile tapers which is transferred to tracing paper.

If the percentile tapers are multiplied by the normal diameter at breast height, 16 inches for example, the result will be actual diameters in inches at the respective percentages of the tree's height above breast height. This is accomplished graphically by shifting the curve in the direction of the diameter axis until the breast-height point ($x = 1.00$) corresponds to the normal diameter in question, 16 inches (Curve II). In this transformation the logarithm of 16 is added to the logarithm of each of the percentile taper series.

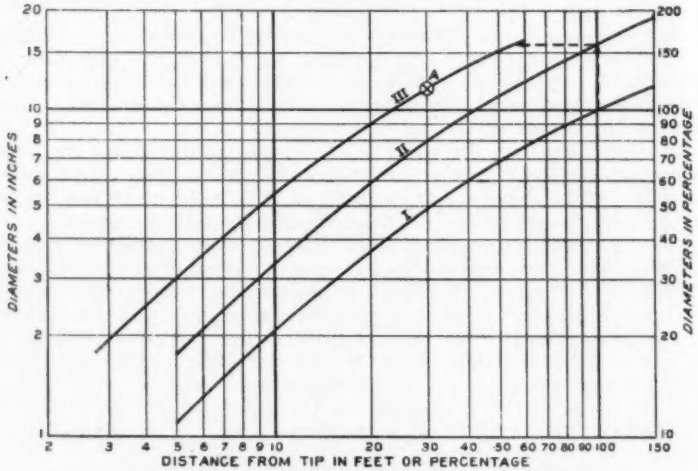

FIG. 24.—Conversion of percentile taper curves to absolute values

To convert the percentile heights at which the diameters are now expressed to actual heights in feet is accomplished in a similar manner by shifting the curve in the direction of the height axis until the breast-height point corresponds to the total height of the tree less 4.5 feet. Thus from Curve III we read the diameter 30 feet from the tip of a 60-foot tree with normal diameter of 16 inches as 11.7 inches. It is necessary to note that the height scale is for distances from tip, so that to read scaling diameters at different heights above ground requires first the simple conversion of the desired heights to distances from tip for each height class.

The conversion of the percentile taper series into tables of taper for different diameter height and form classes may also be accomplished by means of an alinement chart [5] designated to solve the equation of

[5] For a discussion of the theory of alinement charts, see: (*5, 8, 17*).

the stem curve, $y=\frac{x}{a+bx}$. Such a chart has been worked out and is presented as Figure 25. This chart permits direct reading for any value of the form quotient, rather than for certain arbitrary classes, but it is not as convenient to read as the logarithmic chart.

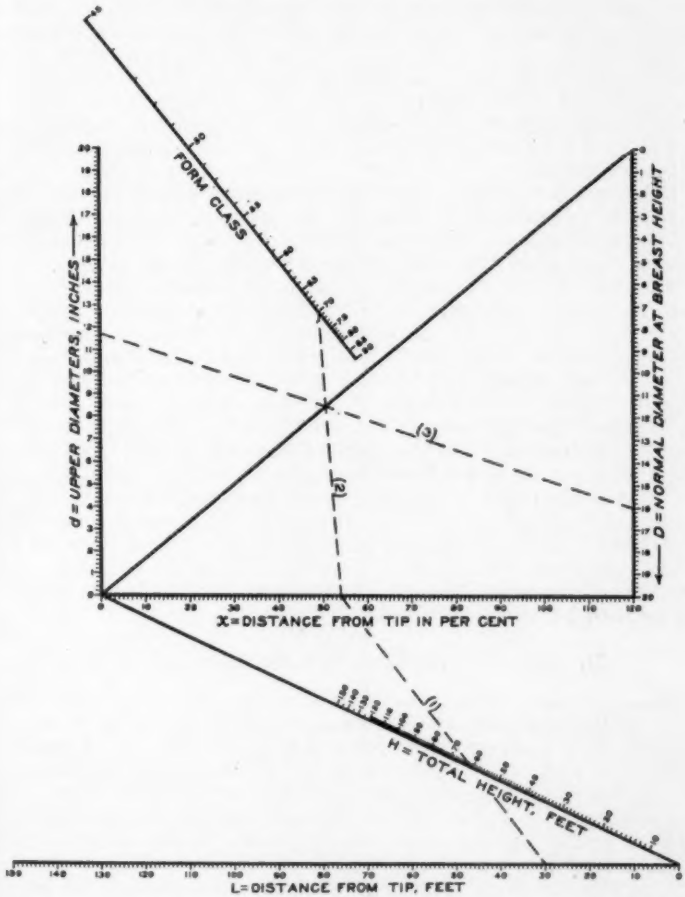
FORM CLASS
d = UPPER DIAMETERS, INCHES
D = NORMAL DIAMETER AT BREAST HEIGHT
X = DISTANCE FROM TIP IN PER CENT
H = TOTAL HEIGHT, FEET
L = DISTANCE FROM TIP, FEET
(1)
(2)
(3)

FIG. 25.—Alinement chart for $\frac{d}{D}=\frac{x}{a+bx}$, giving the diameter at any point on the stem of trees of any diameter, height, and form class

The complete solution of the equation from the chart is best accomplished with the aid of two silk threads attached to small pins at either end, working with the chart fastened upon a drawing board. The equation $\frac{d}{D}=\frac{x}{a+bx}$ may be rewritten in the form $\frac{d}{D}=\frac{x}{1+b(x-1)}$,

since $a+b=1$ in all cases. In this form the equation has four components and can be solved with a pair of intersecting lines on the alinement chart. An extra operation is required, however, to get values for the component x.

The first operation gives the value for x and involves expressing the distance from the tip at which the diameter is to be determined as a percentage of the height above breast height. This is accomplished by stretching one of the strings from the given point on the scale "distance from tip, feet" across the value corresponding to the total height on the diagonal scale for height, to its intersection with the horizontal scale "distance from tip in per cent," at which point a pin should be placed. The broken line (1), Figure 25, shows that 30 feet from the tip of a 60-foot tree is 54 per cent of the distance from tip to breast height.

Now stretch the thread from the pin already placed on the x scale across the required form class on the diagonal scale calibrated for this variable and pin it in this position. Broken line (2) illustrates this for form class 70 and the value of x given by line (1).

Now stretch the second string from a given value on the vertical scale for "normal diameter at breast height," across the intersection of the fixed thread and the unlabeled diagonal line, to the vertical scale for "upper diameters," from which the value sought may be read. In the illustration, broken line (3) shows that the upper diameter 30 feet from the tip of a 60-foot tree of form class 70 is 11.7 inches when the normal diameter at breast height is 16 inches. The upper diameters for the given distance from tip, total height, and form class can now be read for trees of any diameter by rotating the thread for line (3) about the intersection of line (2) and the unlabeled diagonal, and reading the upper diameters corresponding to successive normal diameters at breast height. Just as in the case of the cubic foot volume tables, the relation between actual diameter breast high and normal diameter must be worked out before the chart can be used for any specific table.

II. APPLICATION TO WESTERN YELLOW PINE

The application of the general form-class taper charts and volume tables to the measurement of stands of timber of any given species requires, first, a study of the relation between the normal diameter inside bark at breast height and the actual breast-high diameter for the species in question; second, a study of the variability of the form quotients and absolute form factors of the trees in a stand in order to justify the use of an average form quotient for the entire stand; and, third, a method of determining the average form quotient of the stands to be estimated in the field so that the proper form-class table may be selected for each set of conditions.

These factors, being inherent in the timber itself, enter into the application of any system of volume tables, and a method of analysis which will shed light on any of them will aid materially in improving the technic of timber estimating.

The following sections are not presented with any sense of finality but are merely intended to illustrate the problems to be met and to suggest in the light of work already done—especially that in Sweden, briefly reviewed in the introduction to this paper—what appears to

be the most practical method of handling the application of the general form-class volume tables. Further study is required to give more adequate knowledge of the fundamental principles involved and of the best method of procedure in working out the relations for any specific case, but the results shown for the limited amount of western yellow pine material available are in general agreement with the Swedish work, in so far as they are comparable, and also with fragmentary investigations of other species in this country.

REDUCTION OF BREAST-HIGH DIAMETER FOR DOUBLE BARK THICKNESS AND BUTT SWELL

The simplest method by which the reduction of actual breast-high diameters for bark thickness and butt swell can be taken care of is to determine an average value of the required reduction for each diameter class and apply these figures quite generally over the entire range of the species in question. If it can be shown that the sum of double bark thickness and root swell at breast height is closely enough correlated with the breast-high diameters so that the normal diameters inside bark may be estimated from the latter with a reasonable degree of accuracy, this method will be quite satisfactory for all practical purposes.

Accordingly the sum of double bark thickness and root swell at breast height for each tree was tabulated for the computation of the coefficient of correlation. (Table 12 and fig. 26.) The amount of root swell was in each instance determined as previously described by plotting the original tapers on cross-section paper and extending the normal convex curve of the upper part of the stem down to breast height whenever the root swell extended above this point.

TABLE 12.—*Correlation of double bark thickness and butt swell at breast height to breast-high diameter; western yellow pine*

Double bark thickness and butt swell—Inches (y)	Diameter at breast height—inches (x)																		
	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	44	Total
0.5–0.9	8	5	1																14
1.0–1.4	1	2	16	3	3														25
1.5–1.9		1	6	9	3	2	5	1											27
2.0–2.4				6	6	1	6	3	1	3									26
2.5–2.9				1	2	4	4	2	3				2	1					19
3.0–3.4					1	1		6	2	5	1	1							17
3.5–3.9							2	2	4	2	1	1	1	1	1				15
4.0–4.4							1	1	2	4	1	2	1	3					15
4.5–4.9									5	1				1					7
5.0–5.4							1		1	1			2	1					6
5.5–5.9													3	1	1				5
6.0–6.4										1				1		1			3
6.5–6.9													2	1		1			4
7.0–7.4									1			2							3
7.5–7.9																			
8.0–8.4															1		1		2
8.5–8.9																			
9.0–9.4																1		1	2
Total	9	8	23	19	15	8	19	15	19	17	3	6	11	10	3	3	1	1	190

M_x=16.80 inches; M_y=2.96 inches; σ_x=8.16 inches; σ_y=1.78 inches; r=0.8530±.0198; η=0.865±.0182 b=0.186; S_y=±.9290; y=.186 x −0.16.

From this table the coefficient of correlation was found to be $+0.853 \pm 0.0198$. This correlation appears fairly satisfactory and

seems to indicate that the relation between the breast-high diameter and the corresponding reduction for bark thickness and butt swell may be expressed graphically as a straight line.

To further test this point the correlation ratio corrected for number of arrays[6] was also computed. If the difference between the squares of the correlation ratio and the coefficient of correlation is found to be insignificant, the relation may be assumed to be rectilinear. The correlation ratio was found to be $\pm 0.865 \pm 0.018$.

The difference between the squares of the correlation ratio and the coefficient of correlation is 0.0206 and the standard error of this difference is ± 0.0208. Since the difference is no greater than its standard error it can not be considered significant and a straight line will express the desired relation.

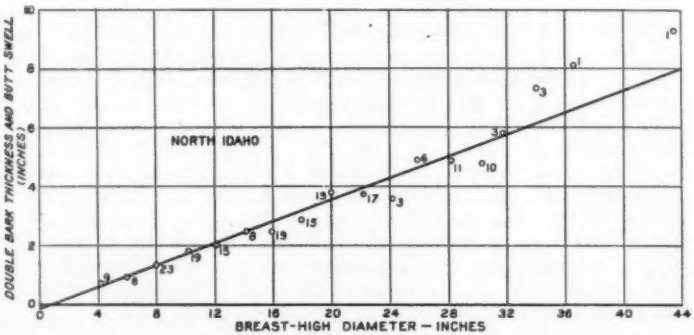

FIG. 26.—Reduction of breast-high diameter for double bark thickness and butt swell, western yellow pine, north Idaho

The formulae and symbols used in these and succeeding calculations are as follows:

d = deviation from approximate mean of variable denoted by subscript

d_o = deviation of approximate mean from true mean

n = number of observations

Σ = sign of summation

m = mean of variable denoted by subscript

σ = standard deviation of variable denoted by subscript $= \sqrt{\frac{\Sigma d^2}{n} - d_o^2}$

r = coefficient of correlation $= \frac{\Sigma d_x d_y - d_{ox} d_{oy}}{n \sigma_x \sigma_y}$

ε_r = standard error of coefficient of correlation $= \frac{1 - r^2}{\sqrt{n}}$

[6] For an explanation of the "correlation ratio" see (22, 27).

σ_{my} = standard deviation of means of y arrays

η_o = correlation ratio = $\frac{\sigma my}{\sigma y}$

η = correlation ratio corrected for number of arrays = $\sqrt{\frac{\eta_o^2 - \frac{k-3}{n}}{1 - \frac{k-3}{n}}}$

k = number of arrays

s_η = standard error of correlation ratio = $\frac{1-\eta^2}{\sqrt{n}}$

$s_{\eta^2-r^2}$ = standard error of $\eta^2 - r^2 = 2\sqrt{\frac{\eta^2-r^2}{n}}$

b = regression coefficient of bark thickness and butt swell on breast-high diameter = $r\frac{\sigma_y}{\sigma_x}$

s_y = standard error in estimating y from x by regression equation = $\sigma_y\sqrt{1-r^2}$

The regression equation for estimating double bark thickness and butt swell from breast-high diameter was computed from the above correlation as $y = 0.186\,x - 0.16$.

The standard error in estimating the double bark thickness and butt swell of individual trees from their breast-high diameter by this relation is ± 0.93 inches. But for volume table purpose we are not interested in the estimation of the normal diameter for each individual tree, but desire only to know the average normal diameter corresponding to each d. b. h. class. The error for such class averages will be reduced in proportion to the square root of the number of trees in the average, so that for 20 trees the standard error would be about 0.21 inches. Since the variability of the observations increases with increase in diameter, and since the error likely to be incurred would be more serious for small trees than large ones, it is desirable to express this error as a percentage of the normal diameter obtained from the estimate, rather than in inches regardless of size as above.

To do this the reduction of breast-high diameter for each tree was computed from the regression equation. These were then compared with the actually measured reductions and the deviations expressed as percentages of the calculated normal diameters. From these percentile deviations, the standard deviation was found to be ± 5.55 per cent. This means that the standard error in calculating the normal diameter for any tree from the regression equation $y = 0.186\,x - 0.16$ will be ± 5.55 per cent.

The same result may be approximated without the arithmetic of computing the deviations for the individual trees by simply expressing the standard error of estimate from the regression equation (0.93 inch) as a percentage of the average diameter of the material (16.8 inches).

To express the error in estimation of normal diameter in terms of effect upon volume we may follow a procedure similar to that used for errors due to variation of upper diameters (p. 710).

Thus for the volume of the tree

$$V=\frac{\pi}{4}Y^2\ Hf$$

$\frac{dV}{dY}=\frac{\pi}{2}YHf$ = rate of change of volume with change in normal diameter.

Since normal diameter, $Y=1$ and since $H=1$,

$V=\frac{\pi}{4}f$, and $\frac{dV}{dY}=\frac{\pi}{2}f$,

$\frac{\pi}{2}f.d$ = change in volume for deviation d.

and

$\frac{\pi}{2}f\sigma_d$ = standard volume error arising from deviation in normal diameter.

Therefore

$$\frac{\frac{\pi}{2}f\sigma_d}{\frac{\pi}{4}f}=2\sigma_d$$ = standard error as percentage of volume due to deviation in normal diameter.

The standard error in percentage of volume corresponding to a standard error ± 5.55 per cent in the estimation of the normal diameter of individual trees will therefore be ± 11.1 per cent.

Of course it is not probable that this relation will prove to be rectilinear for all species, but the general procedure illustrated by the western yellow pine material may still be followed. First, determine whether there is a satisfactory correlation and whether the relation is rectilinear by computing the coefficient of correlation and the correlation ratio. Then fit a curve to the diameter class averages by use of the regression coefficient if rectilinear and graphically if curvilinear.

Further study may indicate the desirability of including some other factor than d. b. h. in the estimation of the double bark thickness and root swell, but nothing definite is available on this point as yet.

USE OF AVERAGE FORM QUOTIENT FOR ENTIRE STAND

The use of an average form quotient for all the trees in a stand is justified by the assumption that positive errors in measuring trees with form quotients below the average will be offset by negative errors in measuring trees with form quotients above the average. This hypothesis will hold good if the form of the trees in the stand is independent of their size and if the average form quotient used corresponds to the average absolute form factor of the trees in the stand and hence to the tree of average volume.

Material from two entirely different types of western yellow pine stands was used to study these points. The first of these stands was typical open virgin growth, partly pure and partly mixed with white fir (*Abies concolor*) and Douglas fir, on good rolling agricultural soil near Worley, Idaho. The second type of stand was fully stocked even-aged second growth, 35 to 55 years old, along the foothills north

of Moscow, Idaho. Forty-eight tree measurements were taken at random from the virgin stand and 47 from the second growth.

The coefficient of correlation between form quotient and diameter was -0.228 ± 0.138 in the virgin stand and -0.453 ± 0.117 in the second growth. These figures indicate practically complete independence between form and diameter in the virgin stand but a slight tendency for the larger trees to have lower form quotients in the even-aged second growth.

The form of the trees within a stand is also practically independent of height, for the coefficient of correlation between form quotient and height was only $+0.259 \pm 0.136$ for the virgin stand and $+0.0245 \pm 0.147$ in the second growth.

These results check with the work of Mattsson, who states that the range of form quotients in trees of the average class is as great as that in the entire stand.

It has previously been shown that the absolute form factors are not proportional to the form quotients, but vary in geometric progression with a ratio of increase of approximately 1.9 per cent per unit of form quotient. However, since this ratio of increase is so

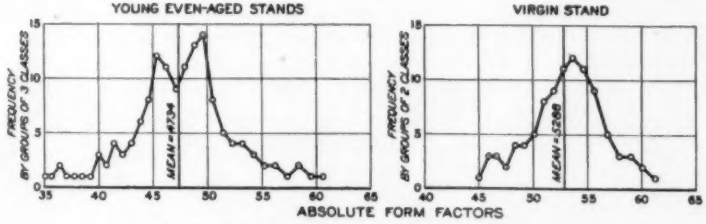

FIG. 27.—Dispersion of form factors in stands of western yellow pine

low, the curve of form factors on form quotients does not depart widely from a straight line within the range of values ordinarily encountered in a stand and so the difference in weight to be given to trees above and below the average form quotient will not be important. Furthermore, the slight error involved from this cause will be offset in some cases because the larger trees tend to average slightly lower in form quotient. Thus the larger trees should be given less weight in an average based on form quotient because of their slightly lower form quotient but more weight because of their greater diameter. For a check on the importance of this point the distributions of absolute form factors and form quotients were analyzed in the two types of stand already described. These distributions are presented in Table 13 and Figure 27. The distributions for both are slightly skew but less for the form factors than for the form quotients. For the virgin stand the actual average form quotient was 74.9 and the form quotient corresponding to the average absolute form factor was 75.1. For the even-aged second growth these figures were 68.6 and 68.8, respectively. The differences are entirely insignificant and the distributions are so nearly normal that we may safely use a direct average of the form quotients and apply the normal laws of error in evaluating the precision of the average.

TABLE 13.—*Dispersion of form quotient and form factors in western yellow pine stands*

YOUNG EVEN-AGED STANDS

Form quotient	Number of trees	Moving average of three classes			Form quotient	Number of trees	Moving average of three classes		
		Median form quotient	Number of trees	Absolute form factor			Median form quotient	Number of trees	Absolute form factor
52.0–52.9		52.5	1	0.351	68.0–68.9	3	68.5	9	0.471
53.0–53.9	1	53.5	1	.357	69.0–69.9	1	69.5	11	.479
54.0–54.9		54.5	2	.364	70.0–70.9	7	70.5	13	.488
55.0–55.9	1	55.5	1	.371	71.0–71.9	5	71.5	14	.496
56.0–56.9		56.5	1	.378	72.0–72.9	2	72.5	8	.505
57.0–57.9		57.5	1	.385	73.0–73.9	1	73.5	5	.514
58.0–58.9	1	58.5	1	.393	74.0–74.9	2	74.5	4	.523
59.0–59.9		59.5	3	.400	75.0–75.9	1	75.5	4	.532
60.0–60.9	2	60.5	2	.407	76.0–76.9	1	76.5	3	.542
61.0–61.9		61.5	4	.414	77.0–77.9	1	77.5	2	.552
62.0–62.9	2	62.5	3	.422	78.0–78.9		78.5	2	.562
63.0–63.9	1	63.5	4	.430	79.0–79.9	1	79.5	1	.573
64.0–64.9	1	64.5	6	.438	80.0–80.9		80.5	2	.584
65.0–65.9	4	65.5	8	.446	81.0–81.9	1	81.5	1	.595
66.0–66.9	3	66.5	12	.454	82.0–92.9		82.5	1	.606
67.0–67.9	5	67.5	11	.462					

VIRGIN STAND

Form quotient	Number of trees	Moving average of two classes			Form quotient	Number of trees	Moving average of two classes		
		Median form quotient	Number of trees	Absolute form factor			Median form quotient	Number of trees	Absolute form factor
66.0–66.9	1	66.0	1	0.450	75.0–75.9	6	75.0	11	0.528
67.0–67.9	2	67.0	3	.458	76.0–76.9	6	76.0	12	.537
68.0–68.9	1	68.0	3	.466	77.0–77.9	5	77.0	11	.547
69.0–69.9	1	69.0	2	.475	78.0–78.9	4	78.0	9	.557
70.0–70.9	3	70.0	4	.483	79.0–79.9	1	79.0	5	.568
71.0–71.9	1	71.0	4	.492	80.0–80.9	2	80.0	3	.579
72.0–72.9	4	72.0	5	.501	81.0–81.9	1	81.0	3	.590
73.0–73.9	4	73.0	8	.510	82.0–82.9	1	82.0	1	.601
74.0–74.9	5	74.0	9	.519			83.0	1	.613

Direct average form quotient:	
Young even-aged stands	68.6
Virgin stand	74.9
Form quotient corresponding to average form factor:	
Young even-aged stands	68.8
Virgin stand	75.1

In the virgin stand the form quotients showed a range of 16 units, from 66 to 82, with a standard deviation of ±3.7 units. In the second growth the range was 28 units, from 53 to 81, with a standard deviation of ±5.6. This higher variability is doubtless due in large measure to the more heterogeneous character of the stands from which the trees are selected. Eliminating only 3 of the most extreme trees reduces the range to 21 units, from 58 to 79, and brings the standard deviation down to ±4.5. In both stands the dispersion followed closely the normal frequency distribution, and the range of form quotients in trees of average diameter was as great as the range in the entire stand. These figures are in close agreement with similar studies made elsewhere, as shown in Table 14, and a standard deviation of ±4.5 in form quotient may be taken as an average for general

use. Since each form-quotient unit corresponds to approximately 1.5 per cent of total volume, the standard volume error in the use of an average form quotient for the entire stand will be ±6.75 per cent.

TABLE 14.—*Dispersion of form quotients within the stand*

Species	Reference	Range				Standard deviation form-quotient units
		Number of trees	Form-quotient units	Minimum form quotient	Maximum form quotient	
Western yellow pine:						
Virgin stand		48	16	66	82	±3.7
Young stand		44	21	58	79	±4.5
Western white pine	Behre (MS8.)	28	22	52	74	±4.5
European larch	Mattsson (1917)		28			±4.4
Jack pine	Wright (1923)	206	28	58	86	±3.9
Northern white pine	do	100	22	59	81	±4.5

ESTIMATION OF AVERAGE FORM QUOTIENT

For the estimation of the average form quotient of stands in the field the indirect method based on the form-point idea which has been developed in Sweden and outlined in the introduction to this paper should be applicable to any conditions. The use of this method requires the working out of a relation between form quotient and form point. For this purpose a procedure similar to that used in working out the reduction of breast-high diameter for double bark thickness and butt swell was employed. Measurements of form point were available for only 138 tress. Of these 48 were taken in different parts of a mature stand of varied density and mixture, 50 were taken on average trees selected from a number of even-aged second-growth stands covering a wide range of ages and sites, and 40 were deduced from measurements of crown length on trees from young second growth and young merchantable timber. The data are tabulated for calculation of correlation in Table 15 and shown graphically in Figure 28.

TABLE 15.—*Correlation of form point and form quotient; western yellow pine*

Form quotient × 100	Form point—(percentage of total length from tip to nearest 0.5)											Total
	13.0–17.5	18.0–22.5	23.0–27.5	28.0–32.5	33.0–37.5	38.0–42.5	43.0–47.5	48.0–52.5	53.0–57.5	58.0–62.5	63.0–67.5	
45.1–47.5											3	3
52.6–55.0								1		1		2
55.1–57.5									1	1		2
57.6–60.0					1	2	3		1			7
60.1–62.5					1	1	3		1	2		8
62.6–65.0				1	1			1				3
65.1–67.5			1	2	1	1	2	2				9
67.6–70.0			2	4	3	2	1					12
70.1–72.5	2	2	3	3	15	1	1					27
72.6–75.0		4	6	13	4	3	1					31
75.1–77.5	2	2	7	8	2	3						24
77.6–80.0		2	3	2								7
80.1–82.5				1								1
82.6–85.0			1	1								2
Total	4	10	23	35	28	13	11	4	3	4	3	138
Actual average form point	15.0	20.9	25.3	29.9	35.3	40.2	44.5	49.4	55.3	59.7	65.0	
Acting average form quotient	74.5	74.9	74.5	73.6	70.4	69.5	64.7	62.4	59.0	57.7	46.6	

The coefficient of correlation was found to be $-.752 \pm .037$ and the correlation ratio, corrected for number of arrays, $-.788 \pm .032$. Although the plotting of the averages, as in Figure 28, indicates some tendency toward curvature, the difference between the squares of the correlation ratio and coefficient of correlation is only 1.38 times its standard error, and as this is hardly sufficient to warrant the assumption of a curvilinear relation a straight-line relationship may be used with sufficient accuracy. The regression equation computed on this basis is—

$y = -.4966 \times +87.22$, which for practical use may be rounded off as

$y = -.05 \times +87$.

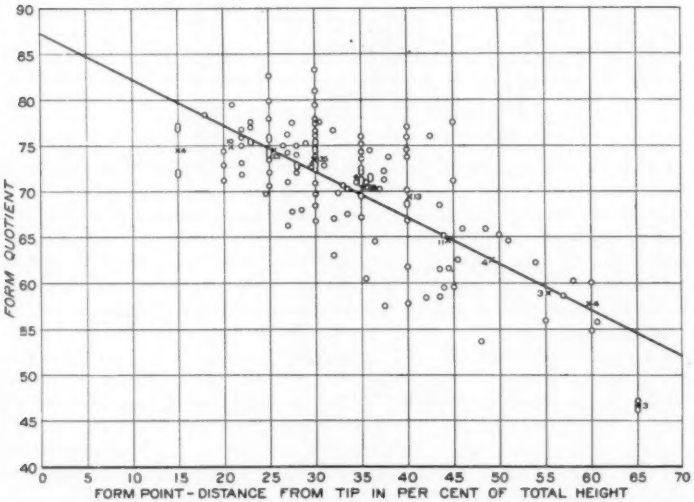

FIG. 28.—Correlation of form point and form quotient; western yellow pine, north Idaho; basis, 138 trees

The standard error of estimating the form quotient of an individual tree from its form-point height by this equation is ± 4.63, which would result in a standard error of cubic volume estimate of about 6.94 per cent. Just as in the reduction of breast-high diameter for double bark thickness and butt swell, however, this relation will be used for estimating the average form quotient of a stand from its average form-point height, rather than for individual tress. The standard error of such an estimate may be derived by statistical method. The standard error in estimating the form quotient of individual trees from their form points (± 4.63 units) is in fact the standard error of the observations of the individual form quotients, the true variability of which has been shown to be about ± 4.5 units. The standard error of the observed values will then be—

$$\sigma_{x_1} = \sqrt{\sigma_x^2 + \sigma_d^2}$$

in which x_1 = observed deviation, x = true deviation and d = error of

observation. From this the standard deviation of an average from quotient based upon measurement of form point on n trees is—

$$\sigma_{ax_1} = \frac{\sigma_{x_1}}{\sqrt{n}} = \sqrt{\frac{\sigma_x^2 + \sigma_d^2}{n}}$$

The number of trees which should be measured as the basis for determination of the average form quotient will therefore depend upon the limit or error which is allowed the average. If the standard error of the average is set at ± 1 form-class unit, which is equivalent to saying that the chances are only 1 to 80 that the observed average form quotient will fall more than half the form-class interval above or below the true average, it will be necessary to measure the form point on more than 40 trees. Thus—

$$\text{If } 1 = \sqrt{\frac{4.5^2 + 4.63^2}{n}},$$
$$n = 41.69$$

The estimation of the average form-point height in the field does not involve any difficulty. It is best accomplished with the aid of a Christen hypsometer calibrated uniformly into tenths so that, when the total height of the tree is just included between the upper and lower notches of the instrument, the relative height of any point on the stem can be read directly as a percentage of the total height. The relative height of the form point is measured with this instrument on a number of trees of normal development selected at random in the stand and these measurements are averaged. The experience of the writer, working with a number of assistants in these stands of western yellow pine and also in timber of other types, confirms the conclusion that different persons will agree so closely in their estimate of form point in the stand that personal judgment offers little difficulty. The writer believes that in practice the forest will not have to be subdivided any further for the application of the form-class principle than is customary in the segregation of types and age classes. In other words, it is thought that a separate estimate of form quotient would only be required for such changes of type or age class as are ordinarily recognized in timber cruising.

The writer also believes that further study of variations in average form under different conditions will make it possible to use, without preliminary estimation, a single form-class table for timber of a given species in certain broad, easily recognized categories throughout its range. His observations in this respect agree with Wright (*26*) who states that all mature stands of northern white pine will fall into form class 70 and that slow-growing black spruce as a type would fall into form class 68. Mature stands of western yellow pine in north Idaho, it is believed, will fall quite consistently into form class 75.

It may be of interest to note that the relation established by statistical method for western yellow pine checks almost exactly with the writer's preconceived notion gained in measuring plots for a yield study in even-aged timber. In this work the average form class for each stand was estimated by eye, and from this the average form point determined. On many occasions a few sample trees were cut and measured in detail. It soon appeared that form class 75 corresponded to a form point 25 per cent from the tip of the tree and that each

increase of 5 per cent in the distance of the form point from the tip corresponded to a decrease of 2.5 form-class units. The statistical result could not have been biased by this notion because most of the trees from which the correlation was computed were measured before the idea had been formulated and in entirely different stands, the form points being measured entirely independently and without any idea of what the form quotients would prove to be.

It is of further interest to compare these results with the form-point relation developed by Jonson (*15*) in Sweden. Jonson's figures fall into a slight curve with form quotients from 3 to 5 units below the western yellow pine regression line. As pointed out in the introduction to this paper Mattsson (*20*) found Jonson's function about 2 units low for European larch, Petrini (*23*) found it equally low for Norway spruce and (*24*) about 6.5 units low for Lapland pine. From the absolute form factors it has been shown that each unit of form class corresponds to a cubic volume difference of about 1.5 per cent. These results suggest that, although it will be necessary to study the form-point relation for each species separately, it may eventually be found that a single function will hold good for many species.

NET ERROR OF ESTIMATE. VOLUME TABLE CHECKS

The method of application of the universal form-class tables having been explained and figures indicative of the errors to be expected in each step in the case of western yellow pine given, it remains to evaluate the net error from all sources which may be expected in the use of this system and to check this theoretical measure of accuracy by tests on representative stands.

Where a species, such as western yellow pine, has a form curve in conformity with the formula errors in estimating volume by the form-class system based on this formula arise from four sources.

In the first place, an error may be expected in the estimate of any tree or stand from the use of an average form quotient for the entire stand. Then an error will arise from the fact that this average form quotient can not be determined by direct measurement, but is estimated indirectly from the average form-point height. The third source of error is the use of an average reduction of the breast-high diameter in each size class for bark thickness and butt swell. Finally, an error is to be expected from the normal variation of upper diameters within the arbitrary form-class interval of five units.

From the foregoing discussion the errors in the estimate of individual trees in any western yellow pine stand are summarized in Table 16.

TABLE 16.—*Standard errors in estimation of volume of individual trees by universal form-class volume table system; western yellow pine*

Source of error	Standard error	
	Unit measured	Equivalent volume
		Per cent
Use of average form quotient for entire stand	±4.5 F. Q. units	±6.75
Estimate of average form quotient from average form point of 42 trees.	±1.00 F. Q. units	±1.50
Estimate of normal diameter from d. b. h.	±5.55 per cent normal diameter.	±11.10
Normal variation of upper diameters within the form-class interval.	±4.2 per cent volume	±4.20
Net error from all sources		±13.74

The net error from all sources is calculated according to the formula—

$$\sigma_s = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2}$$

in which σ_1, σ_2, σ_3, σ_4, are the standard errors from each source separately. This gives a net standard error of approximately 13.74 per cent.

The error in estimating the volume of an entire stand may be derived from these figures by reducing each error proportionately to the total number of trees in the stand—except that the error in the estimation of average form quotient from form point will be the same in the estimate of the entire stand as for any individual tree in the stand unless a greater number of observations are made. On this basis the net standard errors in the estimation of entire stands are given in Table 17 for stand samples of three sizes.

TABLE 17.—*Standard errors in estimation of volume of entire stands by universal form-class volume table system; western yellow pine*

Source of error	Percentage of volume by number of trees in stand		
	100	200	1000
	P. ct.	*P. ct.*	*P. ct.*
Use of average form quotient for entire stand	0.68	0.48	0.21
Estimate of average form quotient from average form point of 42 trees	1.5	1.5	1.5
Estimate of normal diameter from d. b. h.	1.11	.78	.35
Normal variation of upper diameters within the form-class interval	.42	.30	.13
Net error from all sources	2.03	1.78	1.56

From the table it appears that ordinarily the most important source of error in applying this system arises from the estimation of the average form quotient of the stand. Using 40 or more trees for this purpose results in a net standard error in the estimate of stands containing more than 100 trees of less than 2 per cent. Increasing the number of trees upon which the form point is measured to 100, as would be entirely feasible in a strip estimate of a large area, reduces the error to a little over 1 per cent.

To check the above theoretical figures by actual tests on typical samples, the two sets of western yellow pine material already described on page 730 may be used.

The normal diameter for each tree was computed from its actual breast-high diameter with a slide rule. The form height for each tree was read from Figure 23 for the even form class corresponding most closely to the average for the stand. These form heights were then multiplied by the basal areas corresponding to the normal diameters. The total volume was corrected 1.5 per cent for each unit of form quotient by which the actual average of the stand differed from the even form class used in the computation. This result was compared to the actual volume of the trees computed by the Smalian formula for the sections above breast height and with the portion below breast height considered as a cylinder. The deviations of the individual tree volumes corrected in the same way for average form quotient were also computed and averaged.

For the 47 trees from even-aged second-growth stands the actual total volume was 363.66 cubic feet. Their volume was estimated from the formula as described above as 355.87 cubic feet. The difference amounts to −2.14 per cent. The average deviations of the individual trees was 8.81 per cent with the positive and negative errors well distributed over the range of diameter classes.

Since in this check the error in estimating the average form quotient of the stand was eliminated by using the actual average form quotient of the material, the theoretical error to be expected would be derived from the other figures of Table 16 by reducing them in proportion to the square root of the number of trees. The standard error of the aggregate estimate of 47 trees is ±1.95 per cent. On the same basis the theoretical standard error in the estimate of individual trees becomes 13.6 per cent, which is equivalent to an average deviation of about 10.8 per cent. Thus, although the actual aggregate error is a trifle greater than the theoretical standard error, the actual average deviation is less than the theoretical average deviation and the individual errors are well distributed as to sign, so that the check may be considered satisfactory.

The actual volume of 49 trees in the virgin stand was 7,654.35 cubic feet and the estimate from the formula gave 7,722.05, or an aggregate difference of +0.88 per cent. The average deviation of the individual trees was 12.4 per cent. For 49 trees the theoretical standard error is ±1.95 per cent. In this case the actual aggregate error is well within the theoretical standard error, and although the average deviation is slightly greater than the theoretical average error, since the positive and negative errors are well distributed over the range of diameter classes, the check is on the whole satisfactory.

The virgin stand from which these trees were selected was differentiated into two fairly distinct types. About half of the trees came from a shallow draw in which the yellow pine was mixed with other species such as white and Douglas firs. The other half came from adjacent stands in which the timber was pure yellow pine of typical parklike, open character. The trees from each of these types were checked separately to see what difference might be revealed.

In the pure stand 24 trees showed an aggregate error of −7.42 per cent, which is 2.7 times the theoretical standard error of ±2.78 per cent. The average deviation was 11 per cent and the majority of the deviations were negative.

In the mixed type 25 trees showed an aggregate error of +6.77 per cent, or 2.5 times the theoretical standard error of ±2.73 per cent. The average deviation was 14.5 per cent and nearly all the errors were in the positive direction. From these figures it is evident that the mixed stand of virgin western yellow pine differs consistently in some manner from the pure type.

Further examination of the original data indicates that the reason for the large errors in estimating these virgin stands lies in the estimate of normal diameters from the actual breast-high diameters. It appears that in the mixed type the butt swell averaged consistently above the average while in the pure type it ran slightly below the average. In view of this it may be necessary to have separate curves of normal diameter on breast-high diameter for different types or sites; but the available western yellow pine data are not adequate to settle this point.

It is important to note that the failure of the form-class tables, as used in this example, to measure satisfactorily the individual components of the data from the virgin stand of western yellow pine does not constitute a valid argument against the system as a whole. In fact, it serves to illustrate one of the main advantages of the system. It is obvious from the figures given that no single volume table could measure both the pure and the mixed stand accurately, since the differences are inherent in the timber itself and not in the tables used for its estimation. By the form-class percentile taper system these differences can be analyzed and the errors ascribed to their respective causes, whether they be due to differences in the form curve of the trees as a whole, variations in form quotient, or variations in thickness of bark and amount of butt swell at breast height. The author already has under way in spruce and balsam fir in the northeast more detailed studies of the factors influencing bark thickness and butt swell and the variations of form under different conditions.

SUMMARY

I. GENERALIZED TAPER CURVES AND VOLUME TABLES

Unsatisfactory results in the application of volume tables in this country may be due primarily to failure of the tables to recognize differences in form quotient and variations in bark thickness and butt swell at breast height.

A highly satisfactory system of timber estimating has been developed and used in Sweden upon the basis of a formula derived by Höjer to express the general law of stem taper.

In the present studies diameters are measured at intervals of 10 per cent of the height of the tree above breast height. Distortion of butt swell is eliminated graphically from the breast-height measurement, and all diameters are expressed as a percentage of the normal diameter at breast height. Grouping may therefore be done independent of size upon the basis of normal form quotient, which is the ratio of the diameter inside bark half way between breast height and the tip and the normal diameter at breast height.

Analysis of western yellow pine material showed that Höjer's equation gave values for the upper sections of the larger form classes far above those actually attained by the trees. This tendency had also been noted by other investigators, notably in the cases of Scotch pine and European larch in Sweden, and northern white and Norway pines in Canada.

By methods of graphic analysis a new equation, having the form $y=\frac{x}{a+bx}$, was fitted to the western yellow pine material. In this equation x = distance from the tip, expressed as percentage of total height above breast height, at which diameter, y, expressed as percentage of normal diameter at breast height, is measured. A series of tapers fitting this equation will fall into a straight line when values of x/y are plotted on x.

The equation $y=\frac{x}{a+bx}$ was found to fit the following species fairly closely: Norway spruce, Scotch pine, European larch, Siberian larch, western white pine, Douglas fir, Norway pine, northern white pine,

white, red, and black spruce, balsam fir, longleaf pine, shortleaf pine, loblolly pine, and gray birch. For several of these species the values given by the formula are still somewhat higher than those attained by trees of the larger form classes in the upper portions of their stems, but the new formula comes very much closer to approximating the general average form than that devised by Höjer and has the further advantage of being very much easier to work with. Eastern hemlock was the only species differing widely from the formula.

Cubic volume increases geometrically with form quotient with a change of 1.9 per cent of volume above breast height and an average of 1.5 per cent of total volume corresponding to each unit of form quotient. The standard error in percentage of volumes due to the normal variation of upper diameters of the trees included within a form class interval of five units is approximately ± 4.2 per cent.

Comparison of the absolute form factors for the different species with those corresponding to the formula, after making allowance for the standard error to be expected from normal variation of upper diameter within the form class interval (Table 6), shows that for most of the species analyzed the divergence of the tops of the trees from the values given by the formula does not give rise to significant volume errors and that significant errors of more than ± 1 per cent are seldom found except in the larger form classes.

An attempt to modify the basic equation to give a better average fit in the tops of trees with large form quotient gave unsatisfactory results, because no equation could be found which combined a better fit with the simplicity and ease of handling of the equation $y=\frac{x}{a+bx}$. Comparison of absolute form factors with those of an equation of the type $x/y=a+bx+c \log x$, which gives a very close approximation of the average tapers of all the species analyzed, shows that up to form class 75, which is seldom if ever exceeded as an average in forest stands, there is not over 0.5 per cent difference in the total cubic contents as measured by the two equations.

Calculating the tapers of western yellow pine separately by diameter, height, and age, and of red spruce separately by diameter, height, age, crown class, forest type, and site showed no significant differences in the form curve from any of these factors. The most noticeable variation appeared in the red spruce from heavily culled stands and from old-field growth, where the tops fell somewhat further below the values given by the formula than is characteristic for this species under other conditions.

Tables of total cubic-foot volume are constructed from the percentile taper series by the method used by Jonson and outlined in Chapman's "Forest Mensuration". This leads to a set of volume tables based on normal diameter, height, and form class applicable to any species conforming to the formula. Final tables for any individual species are derived from these general tables by reading the normal diameter corresponding to each actual breast-high diameter class for the species in question.

The percentile taper series are easily converted into absolute values for different normal diameters and heights with the aid of logarithmic coordinate paper, or an alignment chart may be used to calculate graphically the diameters at any point on the stem for trees of any diameter, height, and form class.

II. APPLICATION TO WESTERN YELLOW PINE

The application of volume tables derived from the formula by this method requires a calculation of the average reduction of breast-high diameter for double bark thickness and butt swell for different species and an estimation of the average form quotient of each stand.

The average reduction of breast-high diameter for double bark thickness and butt-swell was derived from the coefficient of correlation and may be expressed as 0.186 d. b. h. -0.16. The standard error resulting from estimating the normal diameter of an individual tree from its actual breast-high diameter is ± 5.55 per cent of the desired normal diameter, which is equivalent to ± 11.1 per cent of volume. It is possible that different relations will have to be developed for differences in site or type of timber.

The form quotients of the trees composing any stand are practically independent of diameter, height, and species. The standard deviation of the form quotients within a stand is generally about ± 4.5 units for any species.

Although form quotients are not exactly proportional to absolute form factors, they are so nearly so that an average form quotient may be used safely for the entire stand. The standard volume error in estimating the volume of individual trees from the average form quotient of the stand is ± 6.75.

The form-point system may be used for estimating the average form quotient of the stand. The standard error in estimating the form quotient of individual trees from their form-point heights is ± 4.63 units. Since each unit of form quotient is equivalent to about 1.5 per cent of volume this would result in a standard error of ± 6.94 per cent in cubic-volume estimate.

Measurements of form point on 42 trees in the stand are necessary to attain a standard error in the average form quotient of not over ± 1 form-class unit.

Experience may enable cruisers to judge average form class of stands of timber from the physiognomy of the forest. A separate estimate of the form class should only be necessary for such changes of type or age class as are ordinarily recognized in timber cruising.

The net standard error from all sources in estimating the volumes of individual western yellow pine trees by the universal form-class volume table system is ± 13.74 per cent. In applying this system to entire stands with samples of over 200 trees and average form quotient determined by form-point measurement of at least 40 trees, the standard error is about ± 2 per cent.

CONCLUSIONS

The fundamental principle upon which this system is based, namely, that taper expressed in relative rather than absolute terms is independent of diameter and height for trees of a given form class, was established by European research 17 years ago and has been repeatedly checked since that time.

The further extension of this principle which postulates that taper within a given form class is also independent of species is shown to hold true within certain limits when butt swell is eliminated. Individual species show characteristic differences in taper principally

in the upper portions of their stems, but for most commercial conifers these differences are not sufficient to justify independent treatment.

This permits the use of tables calculated from a mathematical formula and based upon the normal diameter at breast height for a large number of species and eliminates a large part of the labor in volume table construction, at the same time supplying much stronger tables than can be produced by the old system.

The application of these general tables involves a separate study of the reduction of the breast-high diameter for bark thickness and butt swell for each species. This factor is chiefly dependent upon the breast-high diameter itself, but further study is required to discover what other factors may have a measureable influence upon it.

The underlying principle of generalized percentile taper series may be applied with almost equal economy and precision whether or not use is made of a formula for the stem curve. Neither is it necessary to differentiate form classes in the final tables for field application. Tables based on average form for stands of certain characters or for different sites may be derived just as with the old conventional system but without working up an entirely new set of material for each new table. Baker (*2*) has outlined a method of using the percentile curve idea without reference to a formula for the stem curve and without differentiating form quotients in the final tables.

The following are the chief advantages of the generalized percentile taper curve system of volume table construction. Of the points listed, the first four apply especially to the preparation or testing of basic tables; it will require further study to establish completely their validity with respect to the determination of normal diameters at breast height.

1. A relatively small number of trees is required as a basis for dependable tables, because the data are not subdivided into size classes before averaging and therefore fewer trees are needed to get strong averages.
2. The tables are just as strong for extreme values in either diameter or height as they are for size classes well represented in the data.
3. The tables can be extended beyond the range of the data with almost as much certainty as attaches to the main body of the table.
4. The technic of preparing the tables is extremely simple and remarkably free from errors of individual judgment in the placing of curves, etc., numerous graphical checks insuring the accuracy of the arithmetical work.
5. Subordinate tables for different units of product or different standards of utilization may be prepared with a minimum amount of labor.
6. The system provides a basis for analyzing the disturbing factors in the use of volume tables and permits the evaluation of errors from various sources separately. A definite check for similarity of form is provided for testing the application to

any stand of timber and the usual volume checks are, therefore, susceptible of more accurate interpretation. No difficulties in the use of this system have yet been suggested which are not inherent in the timber itself and so equally a source of uncertainty in any system of volume tables. This system gives each factor due consideration, thus bringing the disturbing elements to light and paving the way for improvement of timber estimating technic. Little help can be expected from other systems which ignore these disturbing elements altogether.

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THE EFFECT OF ABSORPTION BY PLANTS ON THE CONCENTRATION OF THE SOIL SOLUTION[1]

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INTRODUCTION

In many irrigated regions the force of circumstances operates in the direction of requiring the most economical and efficient use of irrigation water. In some situations the area of available land is greater than the supply of water. In others, where water must be lifted to reach the land, the cost of pumping is a powerful incentive to sparing use. There are still other situations where it is believed that the extravagant application of water contributes to the saturation of the subsoil, a condition popularly known as water-logging. Many of the troubles that occur on irrigated land are believed to be due either directly or indirectly to the excessive use of irrigation water.

Although there are many cogent reasons for advocating the most sparing use of water in irrigation, there are also reasons why enough water should be used to prevent the accumulation of injurious quantities of soluble salts in the root zone of the soil. Nearly all irrigation waters carry in solution appreciable quantities of salts. Certain of these salts are absorbed by crop plants to a limited extent, but others are used very little if at all. Where the system of irrigation is such that all the water applied to the soil is held within the root zone, the salts brought to the land by the irrigation water remain in the root zone, largely dissolved in the solution.

There are two assumptions with respect to irrigation practice that are very generally accepted but which do not appear to be well supported. These may be stated as follows:

(1) That crop plants absorb the soil solution, including both the water and its dissolved salts, substantially as it occurs in the soil.

(2) That the ideal system of irrigation is one in which only enough water is applied to the soil to moisten the root zone to its water-holding capacity, in order to supply the needs of crop plants and to meet the unavoidable losses of direct evaporation.

These two assumptions are very closely related. The second is really based on the first. If the first is not well founded, the second is certainly open to question.

Persons who have studied the physiology of plants or who are familiar with the literature of that subject do not generally hold the view that plants absorb the soil solution as it exists in the soil. To them the phenomenon of selective absorption is readily accepted as a fact. They conceive that a plant may absorb water or any dissolved electrolytes or gases according to its needs and almost if not quite independently of the relative abundance of the various constituents in the solution. Numerous experiments have been made, the results of which support this view. For the most part these

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experiments have been designed to show the selective absorption by plants as between certain electrolytes in the solution, and it seems generally to have been taken for granted that the rate of water absorption is also independent of the rate of absorption of the dissolved substances. From what is known of the phenomena of absorption by plants it is entirely conceivable that a crop plant might absorb water from a soil solution and at the same time not absorb corresponding quantities of the substances dissolved in that solution. In other words, a plant having its roots in contact with a soil solution might, during its period of growth, modify the character of the solution in the direction of increased concentration—a result similar to that which would follow if water were lost from the solution by evaporation.

Although such a result of absorption by plants is entirely in accord with known facts, there does not appear to be in the literature of the subject very much in the way of direct evidence on this point. For that reason it seemed desirable to conduct experiments that should yield such evidence. The experiments described in the following pages were designed with that end in view. They were planned in detail by the writer but were conducted by James F. Breazeale at the University of Arizona, who prepared and analyzed the culture solutions and who was also responsible for the care of the cultures and for making the various observations, except the conductance determinations, which were made by H. V. Smith, also of the University of Arizona.

PURPOSE OF THE EXPERIMENTS

The experiments were planned to give unequivocal answers to the following questions:

1. Does the plant absorb its nutrient solution as the solution is presented to the roots?
2. If water and electrolytes are absorbed from the nutrient solution at different relative rates, are these rates influenced by the concentration of the nutrient solution?

These questions may be stated in another way:

3. Does the plant, when its roots are in contact with a concentrated nutrient solution, absorb the water and the electrolytes at the same rate?
4. If, even from a dilute solution, the water is absorbed faster than the electrolytes, is the difference in rate of absorption greater as the solution is made more concentrated?

For the information of those who may not have the time or the inclination to read the details of the description of the experiments and the discussion of its results, it may be said here that questions 1 and 3 are answered in the negative and questions 2 and 4 in the affirmative.

DESCRIPTION OF THE EXPERIMENTS

The experiments involved the use of 25 seedlings of barley or wheat, grown as one lot, with their roots immersed in 950 c. c. of nutrient solution. During the period of the experiment the seedlings were supported on a perforated disk covering the glass jar containing the solution. This disk was covered with a soft wax in such a way as to hold the seedlings in position and at the same time to seal the

mouth of the jar and prevent the loss of water from the solution by direct evaporation.

The seedlings were obtained by sprouting a large number of seeds on perforated aluminum disks floating in tap water. When the seeds were well sprouted, with plumules about an inch long, it was possible to pick out lots of 25 that were apparently uniform in size and vigor. These were then transferred to the nutrient solutions and sealed in. The cultures were grown during the latter part of March, 1927, in a greenhouse at the university at Tucson, Ariz.

The culture solutions were obtained in the following manner: A quantity of surface soil taken from a field near Jaynes Station in the Santa Cruz Valley, near Tucson, Ariz., was leached with distilled water through a filter. The percolate was diluted to have a concentration of total salts estimated as approximately 0.5 per cent. This percolate was then analyzed, with the result shown in the second column of Table 2. It was also tested electrically for its specific conductance at 25° C. and for its alkalinity as expressed by the P_H scale. One portion was tested in its natural condition and another portion after it had been boiled and its volume restored by the addition of distilled water.

A part of the original stock solution, found by analysis to contain 5,760 parts per million of total solids, was diluted with an equal quantity of distilled water to obtain a culture solution of one-half that concentration, or 2,880 parts per million. Another part was diluted with 3 parts of distilled water to make a culture solution having 1,440 parts per million. These two diluted solutions were also tested electrically both in the natural condition and after boiling. The results of the electrical tests for all three culture solutions are shown in Table 1.

TABLE 1.—*Total solids, specific conductance, and alkalinity of the three culture solutions before and after boiling*

Solution No.	Total solids (parts per million)	Specific conductance at 25° C.		Alkalinity (P_H)	
		Before boiling	After boiling	Before boiling	After boiling
1	5,760	0.006873	0.007193	8.0	8.0
2	2,880	.003844	.003907	7.9	7.8
3	1,440	.002008	.002075	7.9	7.8

In the experiment of March, 1927, four lots of seedlings of 25 plants each of barley and of wheat were placed in jars of solution No. 3, containing 1,440 parts per million total solids. There were two lots each of barley and wheat in jars of solution No. 2, containing 2,880 parts per million of total solids, and three lots each of barley and wheat in jars of the original solution containing 5,760 parts per million. The seedlings were left in contact with the solution for 11 days, during which period a portion of it was absorbed by the roots and the water was transpired by the leaves. At the end of this period each jar was weighed, and the loss was taken as the measure of the quantity of the solution absorbed and of water transpired.

At the end of this first period also the seal of each jar was broken and a portion of the solution was withdrawn to be tested electrically for conductance and alkalinity, both in its natural condition and after boiling. After these tests the solution that had been withdrawn was then returned to the jar, and enough distilled water was added to replace the quantity that had been transpired. After this addition of distilled water another sample of the culture solution was withdrawn to be tested electrometrically as before. This tested portion was then returned to the culture jar, the jar was again sealed, and the seedlings were allowed to absorb the solution for a period of 13 days.

At the end of this second period of growth each solution jar was again weighed to determine the transpiration loss. The seedlings were then removed from the solution, and each lot was weighed in the fresh condition and again after drying to constant weight at 100° C. The solution remaining in each jar was sampled to be tested electrometrically, following which test each solution was again made up to its original volume by the addition of distilled water and again tested electrometrically. After this test each set of solutions of the same original concentration as used for each kind of seedlings were composited and samples were taken for analysis. In these analyses the six chief constituents were determined for each of the three solution concentrations as shown in Table 2.

TABLE 2.—*Results of analyses of original culture solutions Nos. 1, 2, and 3 and of composites as restored to original volume at the conclusion of the experiment*

[Data in parts per million]

Solids and ions identified	Solution No. 1			Solution No. 2			Solution No. 3		
	Original solution	After barley	After wheat	Original solution	After barley	After wheat	Original solution	After barley	After wheat
Total solids	5,760	5,240	5,280	2,880	2,472	2,456	1,440	1,128	1,200
Calcium	325	291	292	162	147	147	81	66	60
Magnesium	52	44	45	26	17	15	13	9	8
Bicarbonate	112	288	264	56	168	192	28	72	96
Chloride	588	616	588	294	280	266	147	112	112
Sulphate	2,688	2,542	2,545	1,344	1,224	1,205	672	574	570
Nitrate	266	144	160	133	0	0	66	0	0
Total ions identified	4,031	3,925	3,894	2,015	1,836	1,825	1,007	833	846

Table 2 shows that the solutions when restored to original volume were slightly less concentrated at the end of the experiment than at the beginning. It must be realized, however, that the quantity of water absorbed by the seedlings during the 24 days of the experiment was approximately equal to the original volume in each case. This result makes it very evident that the plants did not absorb the solution as it existed in contact with their roots, but that they absorbed chiefly the water and only a small proportion of the dissolved material. In general, each of the ionic constituents was partially absorbed by the plants. The exception was the bicarbonate. With this constituent in every case the quantity identified at the end of the experiment was greater than at the beginning. This increase was probably due to the evolution of carbon dioxide from the plant

roots. The effect of this process was also indicated by a comparison of the P_H values of the solutions at the beginning and at the end of the experiment. The average P_H value of the original solutions was 7.9, whereas at the end it had dropped to 7.6 for the barley solutions and 7.5 for the wheat solutions. This increase in the bicarbonate anion was partly offset by decreases in the sulphate and nitrate anions, both of which were taken up to a greater extent than the chloride anion. The nitrate was completely absorbed from the more dilute solutions.

INTERPRETATION OF RESULTS

The primary aim of this experiment was to observe the effect of the absorption by the plants upon the concentration of the solution. In order to avoid unnecessary complications, it seemed desirable not to use up any of the solution for analysis during the progress of the experiment. Yet it was important to compare the solution concentrations at the end of each growth period with the initial concentrations. For this purpose it seemed practicable to use the method of electrical conductance. With this method a sample of the solution could be withdrawn from each culture jar, its conductance measured, and the sample returned to the jar without appreciable alteration.

In order to standardize these conductance determinations for each solution, each sample that was analyzed gravimetrically for total solids at the beginning and at the end of the experiment was also tested electrometrically. For example, solution No. 1 at the beginning of the experiment showed by evaporation 5,760 mgm. per liter of total solids. The same sample showed a specific conductance at 25° C. of 0.006873 reciprocal ohm. Whence: $\frac{0.5760}{0.006873}=83.8$, or $83.8\times0.006873=0.5760$. At the conclusion of the experiment the three solutions of No. 1 in which barley seedlings had been grown were restored to their original volume by the addition of distilled water and tested for conductance. The mean of these three observations gave a specific conductance at 25° C. of 0.006653 reciprocal ohm. These three solutions were then composited and a sample taken for analysis, which showed total solids of 5,240 mgm. per liter. Whence: $\frac{0.5240}{0.006653}=78.7$. The mean of these two factors, i. e., 81.3, was taken as the conversion factor for interpreting the conductance determinations made on cultures of barley in solution No. 1 during the progress of the experiment. By the same method conversion factors were obtained for each of the other culture solutions.

The detailed interpretation of the observations made during the experiment may be followed through by reference to Table 3, which gives the results obtained from three lots each of barley and wheat seedlings grown in solution No. 1. The volume of solution used for each culture was 950 c. c., and this contained originally 5,472 mgm. of salts or total solids.

TABLE 3.—*Results of growing barley and wheat seedlings in salt solution No. 1*

[25 plants in 950 c. c. culture solution containing 5,472 mgm. of salt]

Growth period and item	Barley seedlings			Wheat seedlings		
	Lot No. 10	Lot No. 11	Lot No. 12	Lot No. 10	Lot No. 11	Lot No. 12
First growth period, 11 days:						
Water transpired—c. c.	232	215	228	241	263	332
Salts—mgm.—Residual [a]	5,105	5,165	5,144	5,159	5,219	5,107
Salts—mgm.—Restored [b]	5,232	5,269	5,246	5,421	5,249	5,202
Mean	5,168	5,217	5,195	5,290	5,234	5,154
Salts absorbed—mgm.	304	255	277	182	238	318
Concentration of original solution—ppm. [c]	5,760	5,760	5,760	5,760	5,760	5,760
Concentration of absorbed solution—do.	1,310	1,177	1,215	755	905	958
Second growth period, 13 days:						
Original salt content—mgm.	5,168	5,217	5,195	5,290	5,234	5,154
Water transpired—c. c.	466	434	406	596	575	639
Salts—mgm.—Residual [a]	4,890	4,906	4,779		4,904	4,553
Salts—mgm.—Restored [b]	5,073	5,168	5,173	5,059	5,127	5,160
Mean	4,981	5,037	4,976	5,059	5,015	4,856
Salts absorbed—mgm.	187	180	219	231	219	298
Concentration of original solution—ppm. [c]	5,440	5,492	5,468	5,568	5,509	5,425
Concentration of absorbed solution—do.	401	415	539	387	381	466
Total growth period, 24 days:						
Total water transpired—c. c.	698	649	634	837	838	971
Total salts absorbed—mgm.	491	435	496	413	457	616
Concentration of absorbed solution—ppm. [c]	703	670	782	493	545	634
Dry weight of 25 plants—gm.	2.47	2.14	2.23	2.44	2.77	2.85

[a] Computed from conductance of residual solutions.
[b] Computed from conductance of restored solutions.
[c] Parts per million.

During the first 11 days of the growth period the barley plants of lot No. 10 transpired 232 c. c. of water, as determined by weighing the culture jar at the beginning and at the end of that growth period. There remained, therefore, 718 c. c. of solution in the culture jar. A sample of this residual solution when tested showed a specific conductance at 25° C. of 0.008747 reciprocal ohm. This value multiplied by the conversion factor 81.3 and the product multiplied by the volume of the residual solution, 718, gives a figure for the quantity of salt left in the solution, viz, 5,105 mgm.

The sample of the residual solution that was tested for conductance was then returned to its culture jar, and distilled water was added to restore the solution to its original volume. After this restoration, another sample was taken for a conductance determination. This sample showed a specific conductance at 25° C. of 0.006774 reciprocal ohm. This figure when multiplied by the conversion factor 81.3 and the product multiplied by the solution volume, 950, gives 5,232 as another figure for the quantity in milligrams of salt in the culture solution. It was deemed advisable to make these two conductance determinations partly as a check against errors of observation and partly because it was thought that the culture solution might be near the saturation point with respect to one or more of its constituent salts, and if this were true the absorption of water by the plants might result in the precipitation of a portion of the dissolved material in the residual solution. The mean of these two results was taken as the best estimate of the quantity of salt remaining in the solution at the end of the first growth period.

The quantity of salt originally in the solution was known to be 5,472 mgm. From this was subtracted the quantity estimated to be in the residual solution, giving a difference of 304 mgm. as the quantity of salt absorbed by the plants. This quantity of salt in relation to the quantity of water transpired, 232 c. c., gives 1,310 mgm. per liter, for the concentration of the absorbed solution. For convenient comparison, this figure is placed in the table in close proximity to that for the concentration of the original solution.

After determining the conductance of the restored solution at the end of the first growth period, the jars were resealed for the second growth period of 13 days. For lot No. 10 the figure for the original salt content for this period was taken as the same as at the end of the first period, 5,168 mgm. During the second period the plants transpired 466 c. c. of water. At the conclusion of this period the quantity of salt remaining in the culture solution was again determined by two conductance readings, and the mean of these was taken as representing the final salt content of the solution. By subtracting this figure from that for the original salt content the quantity absorbed by the plants was obtained, and from this and the quantity of water transpired the concentration of the absorbed solution was computed. Finally the quantity of water transpired for the whole period of growth is given, together with the quantity of salt absorbed, and from these two is computed the concentration of the absorbed solution.

TABLE 4.—*Results of growing barley and wheat seedlings in salt solution No. 2*

[25 plants in 950 c. c. culture solution containing 2,736 mgm. of salt]

Growth period and item	Barley seedlings		Wheat seedlings	
	Lot No. 8	Lot No. 9	Lot No. 8	Lot No. 9
First growth period, 11 days:				
Water transpired ..... c. c.	256	288	328	340
Salts ..... mgm. Residual [a]	[b] 2,524	1,853	2,407	2,516
Salts ..... mgm. Restored [c]	2,746	[b] 2,579	2,459	2,524
Mean	2,524	2,580	2,433	2,520
Salts absorbed ..... mgm.	212	156	303	216
Concentration of original solution ..... ppm [d]	2,880	2,880	2,880	2,880
Concentration of absorbed solution ..... ppm [d]	828	542	924	635
Second growth period, 13 days:				
Original salt content ..... mgm.	2,524	2,580	2,433	2,520
Water transpired ..... c. c.	578	653	712	747
Salts ..... mgm. Residual [a]	2,253	2,249	2,252	.....
Salts ..... mgm. Restored [c]	2,284	2,369	2,278	2,444
Mean	2,268	2,309	2,265	2,444
Salts absorbed ..... mgm.	256	271	168	76
Concentration of original solution ..... ppm [d]	2,657	2,716	2,561	2,653
Concentration of absorbed solution ..... ppm [d]	443	415	236	102
Total growth period, 24 days:				
Total water transpired ..... c. c.	834	941	1,040	1,087
Total salts absorbed ..... mgm.	468	427	471	292
Concentration of absorbed solution ..... ppm [d]	561	454	453	269
Dry weight of 25 plants ..... gm.	2.34	2.65	2.70	2.83

[a] Computed from conductance of residual solutions.
[b] Only one observation used.
[c] Computed from conductance of restored solutions.
[d] Parts per million.

Besides the three lots of barley seedlings grown in solution No. 1, with the results shown in Table 3, three lots of wheat seedlings also were grown in cultures of the same solution concentration, and the results for these are also given in Table 3, Two lots of barley were grown in solution No. 2, containing 2,880 mgm. of salts per liter, for which the results are given in Table 4. With the same solution there were two lots of wheat seedlings, the results from which are also shown in Table 4. Solution No. 3, made by dilution of No. 1 with three parts of distilled water, was used for four lots of barley and four of wheat, the results for which are given in Table 5.

TABLE 5.—*Results of growing barley and wheat seedlings in salt solution No. 3*

[25 plants in 950 c. c. culture solution containing 1,368 mgm. of salt]

Growth period and item	Barley seedlings				Wheat seedlings			
	Lot No. 4	Lot No. 5	Lot No. 6	Lot No. 7	Lot No. 4	Lot No. 5	Lot No. 6	Lot No. 1
First growth period, 11 days:								
Water transpired....c. c....	408	341	267	351	340	337	393	305
Salts....mgm.... {Residual [a]...	1,131	1,219	1,329	1,122	1,253	1,194	1,223	1,276
{Restored [b]...	1,305	1,228	1,263	1,279	1,257	1,207	1,271	1,236
Mean....	1,218	1,223	1,296	1,200	1,255	1,200	1,247	1,256
Salts absorbed....mgm....	150	145	72	168	113	168	121	112
Concentration of original solution...ppm [c]...	1,440	1,440	1,440	1,440	1,440	1,440	1,440	1,440
Concentration of absorbed solution...ppm [c]...	368	425	269	479	333	498	308	367
Second growth period, 13 days:								
Original salt content....mgm....	1,218	1,223	1,296	1,200	1,255	1,200	1,247	1,256
Water transpired....c. c....	671	385	596	681	750	620	723	717
Salts....mgm.... {Residual [a]...	956	1,155	957	995	1,068	1,198	1,119	1,209
{Restored [b]...	1,011	1,110	1,093	1,038	1,096	1,097	1,148	1,184
Mean....	983	1,132	1,025	1,016	1,082	[d]1,097	1,133	1,196
Salts absorbed....mgm....	235	91	271	184	173	103	114	60
Concentration of original solution...ppm [c]...	1,282	1,287	1,364	1,263	1,356	1,263	1,313	1,322
Concentration of absorbed solution...ppm [c]...	350	236	455	270	231	166	158	84
Total growth period, 24 days:								
Total water transpired....c. c....	1,079	726	863	1,032	1,090	957	1,116	1,022
Total salts absorbed....mgm....	385	236	343	352	286	271	235	172
Concentration of absorbed solution....ppm [c]...	356	325	397	341	202	283	211	168
Dry weight of 25 plants....gm....	2.80	1.94	2.36	2.65	2.55	2.42	2.62	2.21

[a] Computed from conductance of residual solutions.
[b] Computed from conductance of restored solutions.
[c] Parts per million.
[d] Only one determination used.

Tables 3 to 5, inclusive, each include the results for individual lots of seedlings, and it is to be expected that they would show a certain diversity, even though the conditions of the experiment were made as uniform as practicable. In order to summarize these results and also to facilitate comparison between the results obtained from the barley and the wheat seedlings when grown in solutions of the same concentrations, the results from comparable individual lots have been averaged and are shown for solution No. 1 in Table 6. This table includes, in addition to the averages of the quantity and concentration of the absorbed solution, a comparison of the average quantity of salts absorbed as determined by conductance and by gravimetric measurements. This comparison gives some indication as to the degree of confidence that may be placed in the final results. A similar comparison of averages for the barley and wheat seedlings

grown in solution Nos. 2, and 3 is also given. These comparisons between barley and wheat show certain differences that may be noted, but probably should not be greatly stressed because the numbers of cultures involved are not large and the differences may not be significant. However, it may be seen that in all three comparisons the wheat seedlings transpired slightly more water than the barley seedlings and also that they produced slightly more dry matter. There are not, however, any consistent differences between the two classes of seedlings with respect to the quantities of salt absorbed.

TABLE 6.—*Average results obtained with separate lots of barley and wheat seedlings grown in culture solutions Nos. 1, 2, and 3, having initial concentrations respectively of 5,760, 2,880, and 1,440 parts per million*

Growth period and item	Solution No. 1 (3 lots averaged)		Solution No. 2 (2 lots averaged)		Solution No. 3 (4 lots averaged)	
	Barley	Wheat	Barley	Wheat	Barley	Wheat
First growth period, 11 days:						
Solution absorbed.....c. c..	225	279	272	334	342	344
Concentration of absorbed solution.....ppm. *..	1,234	873	685	779	385	376
Second growth period, 13 days:						
Solution absorbed.....c. c..	435	603	615	729	583	702
Concentration of absorbed solution.....ppm. *..	452	411	429	169	328	160
Total growth period 24 days:						
Solution absorbed.....c. c..	660	882	887	1,063	925	1,046
Concentration of absorbed solution.....ppm. *..	718	557	507	361	355	231
Salts absorbed, conductance.....mgm..	474	495	447	381	329	241
Salts absorbed, gravimetric.....do....	494	456	388	403	296	228
Dry weight of 25 plants.....gm..	2.28	2.69	2.49	2.76	2.44	2.45

* Parts per million.

Another comparison of the results is shown in Table 7, where the averages for the three sets of barley seedlings are given. Each lot of barley was grown in a solution of different concentration. It will be observed that with a single exception—solution No. 2 for the second growth period—the barley plants absorbed less water from the more concentrated solutions. Also, the concentration of the absorbed solution increased with the concentration of the culture solution in all cases. Correspondingly, the quantity of salt absorbed by the plants increased with the concentration of the solutions.

TABLE 7.—*Average results obtained from growing barley and wheat seedlings in three different solutions, Nos. 3, 2, and 1, having concentrations, respectively, of 1,440, 2,880, and 5,760 parts per million*

Growth period and item	Barley seedlings			Wheat seedlings		
	Solution No. 3	Solution No. 2	Solution No. 1	Solution No. 3	Solution No. 2	Solution No. 1
First growth period, 11 days:						
Solution absorbed.....c. c..	342	272	225	344	334	279
Concentration of absorbed solution.....ppm.*..	385	685	1,234	376	779	873
Second growth period, 13 days:						
Solution absorbed.....c. c..	583	615	435	702	729	603
Concentration of absorbed solution.....ppm.*..	328	429	452	160	169	411
Total growth period, 24 days:						
Solution absorbed.....c. c..	925	887	660	1,046	1,063	882
Concentration of absorbed solution.....ppm.*..	355	507	718	231	361	557
Salts absorbed, conductance.....mgm..	329	447	474	241	381	495
Salts absorbed, gravimetric.....do....	296	388	494	228	403	456
Dry weight of 25 plants.....gm..	2.44	2.49	2.28	2.45	2.76	2.69

* Parts per million.

A similar comparison is made in Table 7 for the three sets of wheat seedlings. In this series there are two exceptions to the rule that more water was absorbed from the more dilute solutions. With but one exception the concentration of the absorbed solution was higher with the more concentrated solutions and also more salt was absorbed by the plants from the more concentrated solutions. There appears not to have been any correlation between the dry weight of the plants and the solution conditions, perhaps because the range of solution concentrations was well within the range to which these plants are able to adapt themselves.

RANGE OF SOLUTION CONCENTRATIONS

In planning this series of experiments it was aimed to keep the concentrations of the solutions well within the limits which occur commonly in the root zone of irrigated soils. It must be kept in mind that the concentration of the soil solution is computed from factors that are different from those used in computing the salt content of the soil. In the latter computation the quantity of soluble material is referred to the dry weight of the soil. Thus when a soil is said to contain 1,440 parts per million of soluble salts, the implication is that for each million pounds of dry soil there are 1,440 pounds of salt. The percentage of water is also usually referred to the dry soil. Therefore, if a soil has a field-carrying capacity for water of 25 per cent, the relationship is such that for each 100 pounds of soil (dry weight) there is held in suspension 25 pounds of water. If in such a soil it were found that each 100 pounds of dry soil contained 0.144 pound of soluble salts, it is assumed that this salt would all be dissolved in the water held in the soil; so that the concentration of the soil solution would be said to be four times that of the salt content of the soil, or 5,760 parts per million.

From this explanation it will be evident that the highest solution concentration developed in these experiments, i. e., at the end of the second growth period of the wheat seedlings in lot No. 12, when the concentration was 15,614 parts per million, or about 1.56 per cent, was not higher than often occurs in the solution of an irrigated soil. A similar solution concentration might be assumed to exist in a soil containing 0.2 per cent of readily soluble salts and about 14 per cent of water. On the other hand, the lowest solution concentration occurring in this experiment, i. e., about 1,300 parts per million, is probably rather higher than that existing in a soil that has been heavily irrigated or saturated with rain.

DISCUSSION OF RESULTS

The results of these experiments appear to warrant the conclusion that crop plants do not absorb water and dissolved substances from the soil solution in the same proportions that these constituents occur together in that solution. With the range of solution concentrations used, and under the conditions of the experiments here described, the residual solutions were definitely more concentrated than the original solutions. The quantity of water absorbed by the plants during the 24 days of the experiments was approximately equivalent to the original volume of the culture solutions, yet at the end of that period

the residual solutions contained 78 to 91 per cent of the salts originally present.

It is true that these experiments were conducted in a laboratory and not in the field, and that plants were grown in culture solutions and not in soil; yet it is believed that the results are fairly applicable to field conditions. Furthermore, there is abundant field evidence to show that when saline irrigation water is used so sparingly that the root zone is seldom or never leached, it is merely a question of time until the soil solution of the root zone becomes so concentrated that crop plants can not absorb from it the water required to make normal growth.

Irrigators are sometimes inclined to believe that if it is necessary to use salty irrigation water it should be used sparingly, so as to avoid carrying to the field more salt than is necessary. In the light of such evidence as is available, this method of procedure is not to be recommended. The only known way to remove soluble salt from the soil of the root zone is by leaching. Consequently, if saline water must be used for irrigation, it should be applied in sufficient quantities not only to supply crop needs and to meet evaporation losses but also to leach the root zone and thus carry away the salts that are left by the water that is evaporated from the soil and absorbed by the plants. In other words, the more salt there is in irrigation water the more water should be used in irrigating. Most of the ordinary crop plants are not able to absorb the water needed for normal growth from a soil solution containing more the 1.5 to 2 per cent of salts. This means that if the irrigation water contains as much as 1,500 to 2,000 parts per million of salt, a sufficient quantity of it should be used so that at least 10 per cent of the quantity applied to the surface of the soil will percolate down through the root zone.

In actual irrigation practice with such water it would be necessary to use more than would be required to insure an average of 10 per cent percolation. The reason for this is that the soil of the root zone of a whole field is seldom uniform in texture or permeability. If only 10 per cent of the water applied were to percolate through the root zone, there would be spots in the field where much less than 10 per cent would percolate; and in those spots there would be grave danger of accumulating excessive concentrations in the soil solution. In order to insure effective percolation and consequent leaching of the root zone, it would be necessary to follow one of two courses—either to apply enough water to the whole field to insure leaching in the areas of least permeable soil, or by a system of interior borders provide a means of holding the water longer on the less permeable areas.

It is not necessary, of course, to use water enough to leach the root zone at every irrigation, particularly where the water is not very salty or when the salts in solution are in part salts of low solubility. With many irrigation waters as much as 60 to 80 per cent of the total salt content consists of salts of such low solubility that they are precipitated from the soil solution before injurious concentrations are reached. This fact needs to be taken into account in estimating what proportion of the irrigation water should be forced through the root zone.

It will be readily understood that in order to leach the root zone the conditions of the subsoil below it must be such that the percolating water may pass away through either natural or artificial

channels. Soils that are underlain by strata of impermeable material such as hardpan, rock, or tight clay, or below which the subsoil is already saturated with stagnant water, can not be leached and consequently are almost certain to become unproductive if irrigated with saline water. This fact makes it essential to take into account the subsoil conditions of irrigable land if the plans for its utilization contemplate its long-continued productivity.

It is coming to be generally recognized that irrigated lands need to have an assured and effective system of drainage, either natural or artificial. Unfortunately, it is not yet so generally recognized that the real function of a drainage system is quite as much to remove surplus salt as to remove surplus water. It is now regarded as normal and routine engineering practice to measure the quantity of water that is delivered to the land for irrigation or that is discharged from irrigated land as drainage, but there are as yet relatively few irrigation engineers who appreciate the need of, or who are equipped to make, determinations of the salt content of irrigation water or of the drainage. With respect to any area of irrigated land there should be available, not only to the farmer but also to the investor, accurate information as to the quantity and character of the salts that are being carried to the land by the irrigation water and also as to the ultimate disposition that is being made of this salt. If the conditions of the subsoil or the methods of irrigation are such that the salt is largely remaining in the root zone, then it is inevitable that the land must ultimately become unproductive. On the other hand, if it can be shown that drainage conditions are such that the salts of high solubility brought in by the irrigation water are being carried away in the drainage, there can be reasonable assurance that the irrigated lands should long continue to be productive.

EXPERIMENTS WITH SODIUM CHLORATE AND OTHER CHEMICALS AS HERBICIDES FOR FIELD BINDWEED[1]

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INTRODUCTION

Field bindweed (*Convolvulus arvensis*) is regarded as the most noxious weed in several of the Western States and is widely distributed in Kansas. In some cases infested farms are reduced in value more than one-half and loan companies often refuse to accept them as security.

Considering the seriousness of the situation and the fact that methods now in use (*5*)[3] in Kansas are not wholly satisfactory, further experimental work seemed desirable. The use of chemicals as herbicidal agents is not new, but so far as is known to the authors sodium chlorate had not been tried in any experiments with bindweed previous to those herein reported.

REVIEW OF LITERATURE

A considerable amount of work has been done by various investigators relating to the eradication of bindweed. Intensive cultivation has proved effective in several States as shown by the following workers: Call and Getty (*5*) in Kansas; Barnum (*3*) and Bioletti (*4*) in California; Cox (*6*) of the United States Department of Agriculture; Stewart and Pittman (*13*) in Utah; and in Idaho (*10*), as reported by the director of the Idaho Experiment Station.

Chemicals have been used more or less generally. Krauss (*12*) and Wilcox (*15*) found sodium arsenite effective. Gray (*7*) also found it effective within the fog belt of California, but not nearly so successful (*8*) in the less humid section of the State. This latter finding was verified by Barnum (*3*). In Idaho (*10*) carbon bisulphide is recommended for use on small areas, but the cost prohibits its general use. Barnum (*3*) in California arrived at essentially the same conclusion. Call and Getty (*5*) and Barnum (*3*) found that salt will kill this weed, but that it has the disadvantage of preventing the growth of all vegetation for an indefinite period. Åslander (*1*) found dilute sulphuric acid of value as a herbicide. The same author (*2*) has recently reported good results with chlorates in the control of Canada thistle. Hutcheson and Wolfe (*9*) found sodium arsenite, nitrate of soda, and common salt to be effective under certain conditions in eradicating hawkweed. Thompson and Robbins (*14*) report results with the use of 37 different chemicals including inorganic

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[2] The cross sections and slides used in making Figures 1 and 2 were prepared by E. C. Miller, professor of plant physiology. The drawings were made by S. Fred Prince, biological artist. The soil samples for nitrification studies were collected and incubated by P. L. Gainey, professor of soil bacteriology, and the sodium arsenite used was furnished through the courtesy of Edgar M. Ledyard, agricultural director of the United States Smelting, Refining, and Mining Co., Salt Lake City, Utah.

[3] Reference is made by number (italic) to "Literature cited," p. 767.

salts, acids and acid formers, gas-forming substances, and oils and other organic substances. Kitchin (*11*) found zinc sulphate and copper sulphate effective in the control of weeds in pine seed beds.

PREVIOUS EXPERIMENTS IN KANSAS

Bindweed was recognized as a serious pest in Kansas as early as 1908, and experiments looking to its control were begun at the Dodge City branch station as early as 1913. These experiments consisted in smothering with a mulch, plowing, hoeing, pasturing with hogs, the use of smother crops, and salting. Salting proved to be the only effective treatment.

In 1915 an infested area at the branch station at Hays, Kans., was fenced and pastured with sheep, but the sheep had to be forced by hunger to graze closely enough to weaken the plants, and the results were so unsatisfactory as to make the method impractical. In 1919 experiments were begun which included different rates of salting, application of fuel oil, intensive fallow, fallow in rotation with smother crops and smother crops alone. Intensive fallow with or without smother crops on large areas and salting the small patches at the rate of 1 pound of salt to the square foot were found to be the only effective methods.

EXPERIMENTAL METHODS

The experiments herein reported were begun in August, 1925, at the Kansas Agricultural Experiment Station, Manhattan, Kans., in a field on which a uniform and thick stand of bindweed had been growing for several years. The soil is a silt loam of the Oswego series, uniform in fertility, and nearly level in topography. This field has been under cultivation for 50 years or more and until 1924 had been used for experimental work in farm crops.

Twelve plots, each 20.87 feet square, or approximately one one-hundredth acre in area, were located in the center of the most vigorous and uniform bindweed growth. They were marked by permanent iron posts driven deep in the ground at each corner. A 2-foot alley was left between the plots. In order to obtain a definite record of the condition of each plot with reference to the stand of bindweed, quadrats 3 by 3 feet were located in each and permanently marked by iron posts. A frame 3 by 3 feet inside dimensions and cross sectioned into square-foot areas was used in obtaining accurate counts of the number of weeds growing in the quadrat.

It was found that some caution is necessary in determining the effect of any treatment on small plots, because of plants sending out underground stems from adjoining plots and alleys. Thus in certain plots in which all plants had been killed a few stray ones were found which had crept in from the adjoining untreated areas. Examination soon disclosed their source and these areas were treated to prevent further intrusion.

It was found by trial that 1 gallon of the solution used was sufficient to moisten the leaves of the plants on a one one-hundredth-acre plot. Since nothing definite was known to begin with as to the concentration and frequency of spraying necessary to kill bindweed, these factors were determined more or less arbitrarily. In deciding

upon the frequency of treatment the growth of the bindweed was taken into consideration. Usually the weeds were treated when they had attained a height of from 3 to 5 inches. In a few cases the cultivation of the fallow plots was delayed by rain. The frequency of treatment is indicated in the tables which follow. The chemicals were applied by means of a 3-gallon knapsack compressed air sprayer.

In order to determine the effect of the various treatments the plants in each quadrat were counted just before each treatment.

EXPERIMENTS WITH DIFFERENT CHEMICALS

The strength of the solution, the time of treatment, and the number of bindweed plants per quadrat before each treatment are shown in Table 1. Except as otherwise noted each plot was treated every time a count of bindweed was made. Thus the time and frequency of treatment indicated in the table by the date on which counts were made. The treatment of the fallow plots consisted of cultivation, and it is, of course, understood that the no-treatment or control plots were not treated in any way. The number of plants, however, was determined on the control plots whenever they were counted on the other plots.

TABLE 1.—*Number of bindweed plants per quadrat in fallow and untreated plots and in plots treated with different chemicals; August 18, 1925, to May 1, 1927*

Date of treatment	Number of bindweed plants per quadrat											
	Plot 1; copper sulphate, 12.5 per cent	Plot 2; fallow	Plot 3; zinc chloride, 12.5 per cent	Plot 4; sodium arsenite, 2 per cent	Plot 5; no treatment	Plot 6; sodium chlorate, 25 per cent	Plot 7; sodium chlorate, 12.5 per cent	Plot 8; fallow	Plot 9; sodium arsenite, 1 per cent	Plot 10; zinc chloride, 6.25 per cent	Plot 11; no treatment	Plot 12 copper sulphate, 6.25 per cent
1925												
Aug. 19	31	27	44	25	22	27	35	25	43	65	33	41
Sept. 2	25	25	35	23	23	23	24	35	28	45	30	27
Sept. 16	40	42	47	38	17	[a] 19	[a] 10	58	30	47	28	29
Sept. 29	54	56	56	47	23	4	4	36	46	59	32	31
1926												
Apr. 30	70	77	67	[b] 65	49	0	0	50	[b] 73	79	101	45
May 11	67	75	77	77	------	0	0	49	81	112	------	60
June 2	99	91	93	88	------	0	0	49	84	98	------	60
June 14	67	91	71	66	------	0	0	51	61	90	------	50
June 30	------	88	------	------	------	------	------	48	------	------	------	------
July 12	75	76	66	58	------	0	0	48	54	71	------	40
July 26	------	92	------	------	------	------	------	30	------	------	------	------
Aug. 8	58	44	47	29	------	------	------	23	------	------	------	45
Aug. 17	------	89	------	------	------	0	0	30	38	43	------	------
Aug. 30	------	123	------	------	------	------	------	34	------	------	------	------
Sept. 18	57	118	32	20	------	0	0	31	42	52	------	59
Oct. 19	75	98	46	24	------	0	0	22	59	58	------	62
1927												
Apr. 30	68	76	61	35	------	0	0	30	46	49	------	v

[a] Plots 6 and 7 were not treated after Sept. 16, 1925.
[b] In 1926 the strength of spray on plots 4 and 9 was increased to 4 per cent and 2 per cent, respectively.

It will be noted that none of the treatments except that with sodium chlorate was effective in decreasing the number of bindweed plants. On the contrary there seems to have been a marked increase in numbers in some cases. Sodium arsenite and zinc chloride killed the tops of the plants and at first appeared to be very effective, but

new growth appeared soon after each treatment and the roots seemed to be uninjured. Copper sulphate had little effect except the fourth application, which killed almost all top growth. This application was made on September 29 and may have coincided with a critical period in the life of the plants. This result suggests the need of more information regarding the relation between stage of growth and susceptibility. In all cases except possibly the sodium arsenite, the weaker solutions seemed to have been about as effective as the stronger. At the suggestion of the manufacturers the strength of the solution of this chemical was increased in 1926. The effectiveness of the treatment was somewhat increased, but it did not approach the sodium chlorate in this respect. It will be noted that it was necessary to cultivate the fallow plots more frequently than it was necessary to spray the chemically treated plots.

Sodium chlorate was markedly and quickly effective, only three applications being necessary to kill all the plants. The effect on the appearance of the plants was also very different from that produced by the other herbicides. Its action at first appeared to be slow. Comparatively little effect was noticed at first, but gradually the plants became pale green or yellow, the leaves began to curl, and finally the entire plant withered and died. Each succeeding day showed more and more dead and dying plants. The older leaves died first, the reaction progressing toward the tip of the vines. After the second application it was observed that the vines were slender and assumed an upright position instead of the normal vigorous and prostrate growth.

On September 29—that is, at the time of the fourth application for other treatments—the plants on the sodium chlorate plots had been so reduced in number and vigor that further treatment was considered unnecessary. As a matter of fact it is doubtful if the third treatment on September 16 was necessary, since the plants at that time were obviously weak and were rapidly dying. On September 29 the roots of the bindweed plants on the various plots were examined under the microscope. Those from the plots treated with sodium chlorate had a dark ring around the wood portion which was not observed in those from other plots. There was also a noticeable absence of new shoots on the roots. These conditions prompted a more detailed microscopic and chemical examination which was made at the time the growth ceased in the fall.

On September 30, twenty or more plants in the sodium chlorate plots which because of special vigor had previously been marked for observation, were found to be making no growth but were turning yellow and becoming more sickly in appearance. On October 5, approximately three weeks after the last treatment, no increase in growth was apparent, the plants being dead or dying. New plants reappeared from time to time, only to fade away in the course of four or five days.

A very heavy white frost occurred on October 10, the minimum temperature reaching 25° F. Observations were made as soon as the frost had disappeared and again three days later. Bindweeds surrounding the experimental plots were damaged very little, if any. This was also true of all the experimental plots except those which had been treated with sodium chlorate. These, however, had been

severely injured, most of them being frozen to the ground. This observation seems to be especially pertinent in view of the fact that untreated plots were later observed to have survived temperatures of 14° F. and the tops were not killed until late in November. This would seems to indicate that bindweed is not sensitive to frost as is commonly believed.

ROOT STUDIES

The gradual killing of the plants by sodium chlorate and their susceptibility to freezing, as previously noted, suggested a study of the roots. Accordingly, on October 15, roots were taken from the various plots and cross-sections were made and examined. Important differences were noted in the starch content of the cells and in the condition of the cell walls. The cells in the roots from the untreated plots were filled with starch granules and the cell walls were intact. On the other hand, roots from the plots treated with sodium chlorate were characterized by the absence of starch granules, and the cell walls in many cases showed unmistakable signs of disintegration. An attempt is made to show this condition in Figures 1 and 2, which are camera-lucida drawings of cross sections of the roots from the various plots. The heavily shaded areas within the cells represent stored starch. The light shading between cells represents disintegration of cell walls.

It will be noted that the condition of the roots as represented in the figures corresponds closely with the growth and condition of the plants as previously discussed.

Just why sodium chlorate affects the plants as it does is not known. Observations indicate that it interferes with photosynthesis and compels the plant to draw upon the food reserve in the roots, until the supply is exhausted and death occurs.

STRENGTH OF SODIUM CHLORATE SOLUTION AND FREQUENCY OF APPLICATION

The fact that the 12.5 per cent solution of sodium chlorate was as effective as the 25 per cent solution suggested the possibility of using even weaker solutions. It was also thought that the bindweed plants might be more susceptible at certain periods of growth and that the time of treatment might be an important factor. Experiments were therefore started in 1926 to determine the best time for treatment, the minimum number of treatments, and the minimum strength of solution that could be used effectively. Eleven additional plots were laid out and treated as indicated in Table 2. As in the preceding experiments the number of bindweed plants was determined just before each treatment, except as otherwise stated. The results are presented in Table 2.

In general the treatments in 1926 seemed to be not quite as effective as those of the preceding year, owing it is believed to a severe drought, which retarded the growth of the bindweed and hence afforded less opportunity for exhaustion of the root reserves, which, as previously explained, seems to be a significant factor in control with sodium chlorate. Nevertheless the results in a broad way agree with those of the preceding year and afford additional proof of the effectiveness

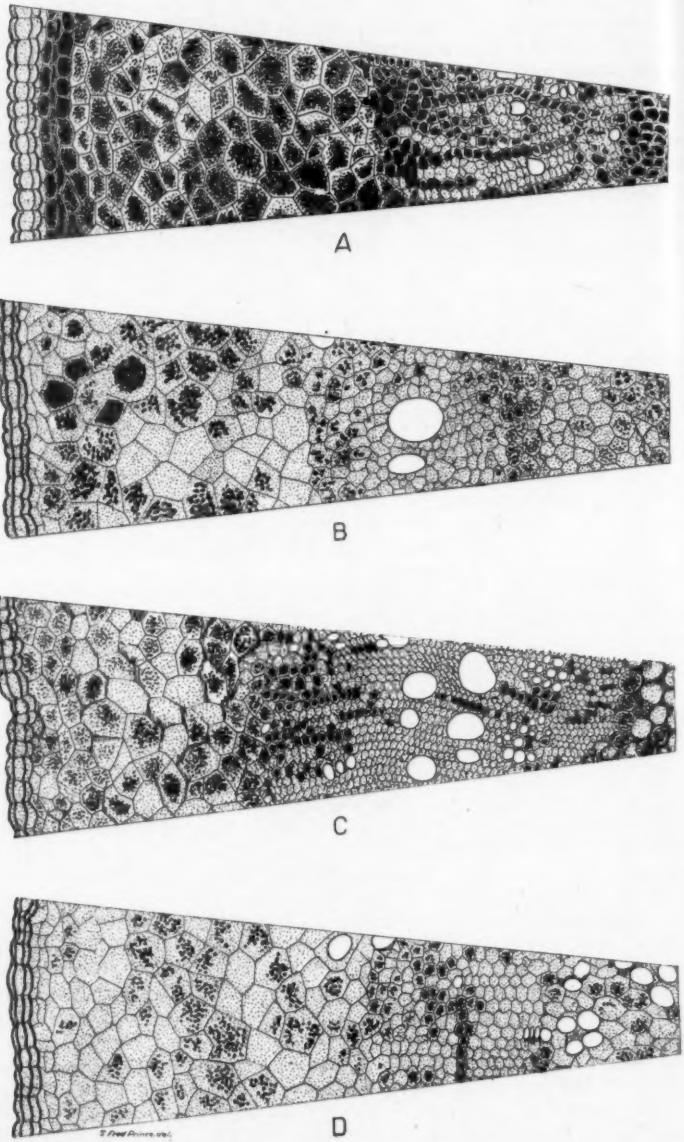

FIG. 1.—Cross sections of roots of bindweed from plots receiving: A, no treatment (control); B, copper sulphate treatment; C, fallow treatment; and D, zinc chloride treatment

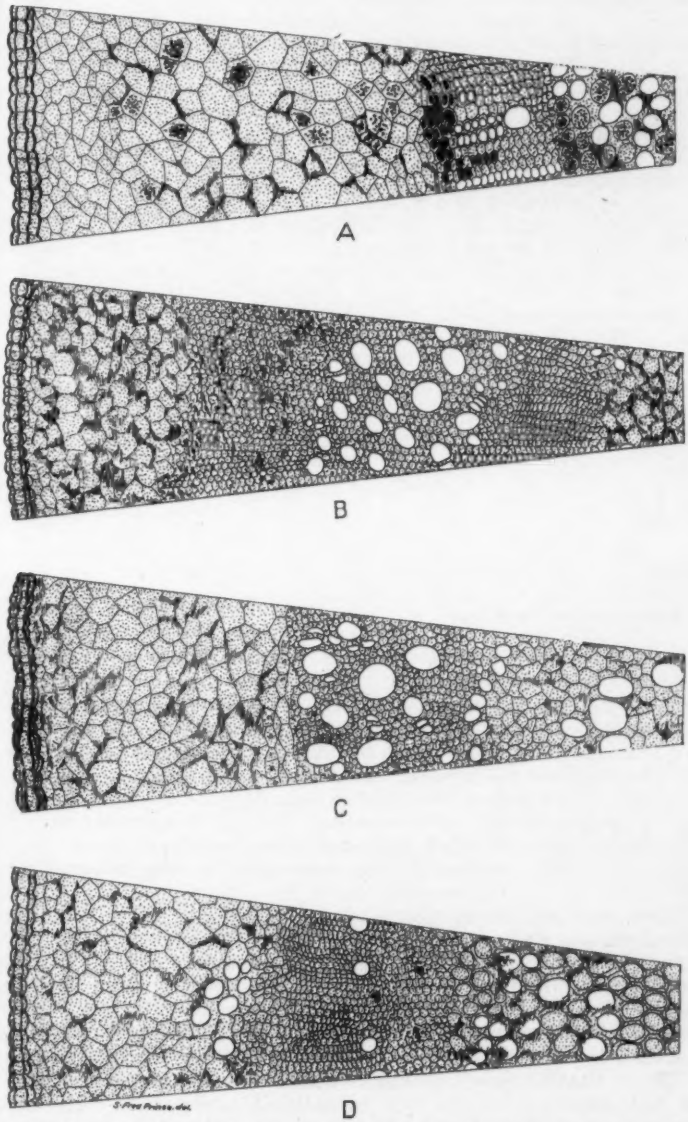

FIG. 2.—Cross sections of roots of bindweed from plots treated with: A, sodium arsenite; B, sodium chlorate (plant with top dead to crown); C, sodium chlorate (plant with top frosted to crown; and D, sodium chlorate (plant with top green)

of sodium chlorate. They also afford information as to the proper time and frequency of treatment.

TABLE 2.—*Number of bindweed plants per quadrat in plots treated on different dates with sodium chlorate solution of different strengths*

Date of treatment	Number of bindweed plants per quadrat										
	Plot 5, 12.5 per cent solution	Plot 11, 12.5 per cent solution	Plot 17, 6.25 per cent solution	Plot 20, 3.125 per cent solution	Plot 21, 12.5 per cent solution	Plot 22, 12.5 per cent solution	Plot 23, 12.5 per cent solution	Plot 24, 12.5 per cent solution	Plot 25, 12.5 per cent solution	Plot 26, 12.5 per cent solution	Plot 27, 12.5 per cent solution
1926											
Apr. 30			28	73							
May 14			59	63	29	43	65				
June 2			57	48				63			
June 14			60	50			[a] 36	[a] 27			
June 17					37	33					
June 23									40		
July 8			49	57	25	33		[a] 43	[a] 6		
July 13							39			47	
Aug. 2								62			
Aug. 5			57	53	[a] 39	[a] 47					
Aug. 17	71	86							48		
Aug. 30										66	
Sept. 15			48	54	38	44					
Sept. 19		79									
Sept. 22											52
Oct. 29 [a]	47	7	13	32	21	23	42	35	32	47	34
1927											
Apr. 30 [a]	26	17	0	3	0	0	10	2	3	14	18

[a] Weeds counted but no treatment.

It appears that little or nothing is gained by applying sodium chlorate before the plants are approximating the bloom stage. For example the treatment of plot 24, which was sprayed on June 2 when the plants were in full bloom and again on August 2, was more effective than the earlier treatment of plot 23, and almost as effective as the four applications on plots 21 and 22 which were first treated on May 14. Likewise the three applications to plots 6 and 7 of Table 1, beginning August 19, 1925, was but slightly more effective. The treatment of plot 25, which consisted of two applications, the first of which was made on June 23, was essentially as effective as that of plot 24.

When the first application was not made until after July 1, the treatment was much less effective than when treatments began in the bloom stage. This is illustrated by plots 11 and 26 as compared with plots 24 and 25. The reason perhaps is the severe drought, which, as previously mentioned, prevented normal growth. From a practical standpoint applications should not be delayed sufficiently to allow a seed crop to mature.

The results from plots 17 and 20 indicate that the weaker solutions are not effective enough for economical eradication. Thus two applications of the stronger solution as on plots 24 and 25 were as effective as the seven applications of the weaker solutions on plots 17 and 20.

A practical demonstration of the effectiveness of sodium chlorate in eradicating bindweed was made by the department of horticulture of

the Kansas State Agricultural College in 1926. A roadway 100 rods long through an orchard bordered by a heavy growth of bindweed was treated three times beginning at the period of full bloom. The weeds were completely eradicated.

EXPERIMENTS WITH SODIUM HYPOCHLORITE

The successful experiments with sodium chlorate suggested the possibility of using a cheaper material, which would react in a similar manner. An opportunity to conduct such experiments presented itself in the spring of 1926 and accordingly five additional plots were laid out. One of these was treated with "K. M. G. weed killer" and the others with 1, 2, 3, and 4 per cent solutions, respectively, of sodium hypochlorite. The results are presented in Table 3.

TABLE 3.—*Number of bindweed plants per quadrat in plots treated with sodium hypochlorite or with K. M. G. weed killer; April 30, 1926, to May 2, 1927*

Date of treatment	Number of bindweed plants per quadrat				
	Plot 19; sodium hypochlorite, 1 per cent	Plot 18; sodium hypochlorite, 2 per cent	Plot 13; sodium hypochlorite, 3 per cent	Plot 16; sodium hypochlorite, 4 per cent	Plot 15; K. M. G. weed killer
1926					
Apr. 30	30	39	53	80	70
May 11	48	64			
June 2	36	53	46	61	30
June 14	55	56	47	78	43
July 12	32	62	50	72	48
Aug. 8					52
Sept. 18	26	41	45	57	57
Oct. 19	33	56	46	58	54
1927					
Apr. 30 [a]	42	23	14	52	66

[a] No treatment.

The earlier applications of sodium hypochlorite were regarded as of little value because of the difficulty in preparing this material and the lack of uniformity in concentration. Later in the season it was prepared by using liquid chlorine. In this way it was possible to control the process and make a uniform product. The last two applications were made with material prepared in this way. The stronger solutions killed many of the bindweed, but none were as effective as the sodium chlorate. It is thought that still stronger solutions will be more effective, and future work will include such solutions. The effect on the appearance of the bindweed is very similar to that produced by sodium chlorate; that is, the plants die very slowly, assuming first a sickly yellow appearance followed by gradual dying of the leaves and finally by death. K. M. G. weed killer had no appreciable effect in reducing the number of bindweed.

EFFECT OF HERBICIDES ON THE SOIL

In view of the effectiveness of sodium chlorate in eradicating the bindweed it seemed desirable to determine whether it and other herbicides had any deleterious effect on the soil. Accordingly 11 of

the plots were sampled to a depth of 7 inches in May, 1926, and the nitrifying power of the soil determined. The results are presented in Table 4.

TABLE 4.—*Milligrams of nitrate per 100 gm. of soil in plots receiving different treatments*

Plot No.	Treatment	Milligrams of nitrate (NO_3) per 100 gm. of soil			
		Present when sampled	Formed during incubation		
			With water	With water and ammonium sulphate	With water and cottonseed meal
15	K. M. G. weed killer	0.88	8.78	29.48	52.24
16	Sodium hypochlorite, 4 per cent	1.95	3.72	4.48	15.15
1	Copper sulphate, 12.5 per cent	.62	7.05	59.04	70.21
2	Fallow	5.15	6.65	37.35	65.68
3	Zinc chloride, 12.5 per cent	3.24	7.94	32.17	83.94
4	Sodium arsenite, 10 per cent	7.46	11.96	86.98	59.21
5	Control	.44	21.35	84.56	105.81
6	Sodium chlorate, 25 per cent	24.96	13.61	23.36	69.48
17	Sodium chlorate, 6.25 per cent	7.08	11.81	16.53	28.33
18	Sodium hypochlorite, 2 per cent	4.53	10.38	13.18	37.97
21	Sodium chlorate, 12.5 per cent	4.72	7.60	11.02	15.21

It will be noted from these results that the soil from all plots contained nitrates when sampled. When these samples were incubated additional nitrates were formed, showing that bacterial action had not been seriously interfered with by the various treatments. There is some evidence of a retarding influence of the treatment in case of plots 16, 17, 18, and 21. These plots were treated with sodium chlorate and sodium hypochlorite a short time before the soil samples were taken. The effect is particularly noticeable where ammonium sulphate and cottonseed meal were added before incubation. That the retarding influence is temporary is indicated by the fact that it is not apparent in plot 6 which had received a heavy application of sodium chlorate the previous fall.

Plots 6 and 7 on which bindweed had been killed by sodium chlorate, were prepared and sown to wheat in September, 1926. This crop made a normal growth throughout the fall and came through the winter in excellent condition. So far as could be observed the sodium chlorate had no deleterious effect upon the growth of wheat.

SUMMARY AND CONCLUSIONS

Experiments to determine the relative effectiveness for the eradication of field bindweed of fallow, sodium chlorate, sodium arsenite, sodium hypochlorite, zinc chloride, copper sulphate, and K. M. G. weed killer were conducted. Various concentrations were used and observations were made as to the frequency and time of application. The roots were examined microscopically, drawings were made, and food reserves were studied. The results indicate that sodium chlorate is unusually promising for the eradication of field bindweed.

The optimum time for the application of the first spray is probably about the time the plants are in full bloom.

There seemed to be no permanent detrimental effect upon the soil from any of the chemicals used in these experiments.

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